THE EFFECT OF ACCUMULATION ON TEMPERATURES WITHIN A SNOWFIELD

By A. E. Benfield

Lecturer on Applied Science, Harvard University

Abstract. The wave of cold temperature, which begins to propagate into a snow field during the winter, may be considerably enhanced by the thermal blanketing of snow accumulating on the surface. This effect is discussed theoretically by means of calculations based on the mathematical theory of the conduction of heat in solids. The results of these calculations show that for an assumed steady velocity of accumulation at the rate of 4 meters/year or 8 meters/year, the winter cold wave should assume significantly larger proportions than in the absence of accumulation.

Zusammenfassung. Die Welle tieferer Temperatur, die im Winter in ein Schneefeld eindringt, kann durch den Effekt des sich an der Oberfläche ansammelnden Schnees beträchtlich gesteigert werden. Dieser Effekt wird theoretisch nachgewiesen mit Hilfe von Berechnungen, die auf der mathematischen Theorie der Wärmeleitung in festen Körpern beruhen. Die Ergebnisse dieser Berechnungen zeigen, dass die Wintertieftemperaturwelle, eine gleichmässige Schneeansammlung von 4 Metern/Jahr oder 8 Metern/Jahr angenommen, bedeutend grössere Beträge annehmen würde als beim Ausbleiben solcher Ansammlungen.

Introduction

The thermal conditions within the firn and ice of a glacier have been experimentally investigated in some detail by a number of workers during the past quarter of a century [1, 2, 3]; the temperatures existing below the surface of a glacier have been found to fluctuate with the time of year and to depend on a number of factors. One factor which may often be of minor importance but which, in a region where the winter's accumulation of snow is large, is capable of producing a considerable effect on the sub-surface temperatures, is the thermal blanketing effect of accumulating snow on the layers which it covers.

The purpose of this paper is to show quantitatively, by means of calculations based on the mathematical theory of the conduction of heat in solids, to what extent this blanketing effect may be expected to lower the sub-surface temperatures.

In the normal course, whether or not the accumulation is of a significant amount, a cold temperature wave becomes established in the upper layers of the firn or ice during the winter months. In the absence of any melt water, rain or the transfer of an appreciable amount of heat within the firn by liquid water or by radiation, this cold temperature wave penetrates slowly downward as the weeks go by, in a regular manner. During the cold part of the year, when no liquid water penetrates the firn, it has been shown by measurements that the actual temperature distributions are in close enough agreement with those calculated from the laws of thermal conduction to establish that the temperature within the firn during these periods is largely controlled by thermal conduction alone. Later, when the temperature at or above the surface becomes warm enough to produce liquid water, the sub-surface thermal conditions become rapidly and radically changed; thermal conduction no longer exerts a controlling influence. However, we are not concerned in this paper with those relatively rapid changes of temperature which occur when melt water is present, but only with the thermal situation during the preceding period, when thermal conduction is the dominating factor.

Whilst the laws of thermal conduction apply, the downward velocity of penetration of the cold temperature wave is determined by the thermal constants of the material and by the period of the fluctuation of the temperature at the surface. As is well known, the amplitude of this temperature wave in the firn decreases with depth. However, as illustrated below, if the accumulation of snow during the winter is large enough to raise the surface of the snow field at a rate which is comparable with the downward velocity of penetration of the cold temperature wave, the blanketing effect of the successive layers not only effectively buries the cold wave to a greater depth than would occur in the absence of accumulation, but it can also considerably enhance the magnitude of the wave, so that significantly lower temperatures penetrate to greater depths in the firn than would otherwise take place.

A region where accumulation may become large enough to make its effect felt in this way is the upper Seward Glacier,[4] Yukon Territory, Canada, where temperature measurements have recently been made [5] with a view to studying the thermal regimen of the firn. Large accumulations of snow, sufficient to cause an appreciable thermal effect of this type, can also occur on the Jungfraujoch and elsewhere.

The mathematical problem of calculating the effect of accumulation involves a consideration of the heat-flow in a moving medium; it has been solved [6] for the case of a steady rate of accumulation and constant thermal properties of the material. As the solution of the problem, even with these simplifying assumptions, is necessarily somewhat mathematical and involved, and has been given before,[6] we shall describe here merely the results of calculations based on the solution, that is, the magnitudes of the effect to be expected for specified conditions, at certain times of year and for different rates of accumulation, for the benefit of those who may not care to follow the mathematical solution, or to go to the considerable labor of applying numerical values to it.

Discussion of the Calculations

For the purposes of calculation, the annual temperature variation at the surface of the snow field has been assumed to be a cosine wave of amplitude 10° C., fluctuating between limits of 0° C. and −20° C. A graph of this surface temperature wave, plotted against time, is shown in Fig. 1 (p. 252). The amplitude, and similarly the limits of the fluctuation of the surface temperature, were arbitrarily chosen; other values for these temperatures might equally have been used.

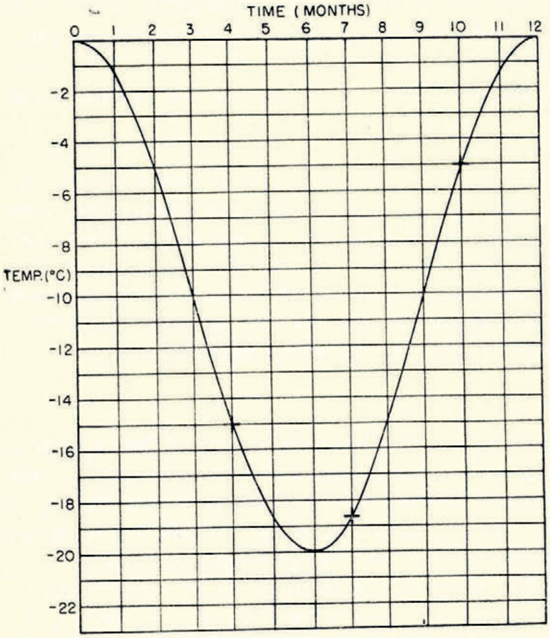
Any actual annual temperature variation is not, of course, a mathematically smooth cosine curve of the type of Fig. 1. However, irregularities and departures from a cosine curve may be analysed by the method of Fourier into components of higher frequency, and treated by the very same methods which are applicable to the fundamental cosine curve. Now, the effect of these components of higher frequency (and thus also of departures from the cosine curve) attenuates with depth more rapidly than that of the fundamental; hence, the components of higher frequency have a correspondingly small influence on the sub-surface temperatures; their influence may often be negligible compared with the temperature disturbance created by the fundamental alone.

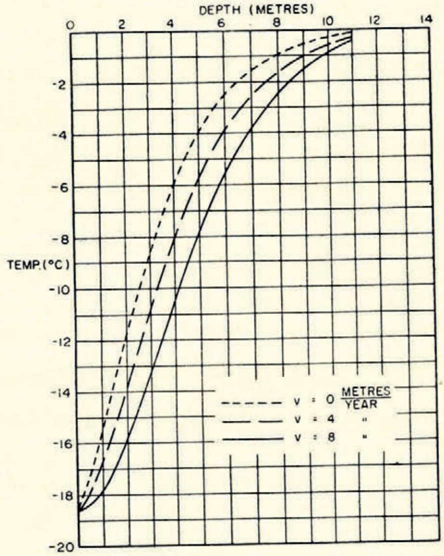
The snow field is assumed to be of infinite depth and extent. In practice, no serious error is introduced by these assumptions for a snow field whose depth and linear horizontal dimensions are large compared with 15 meters.

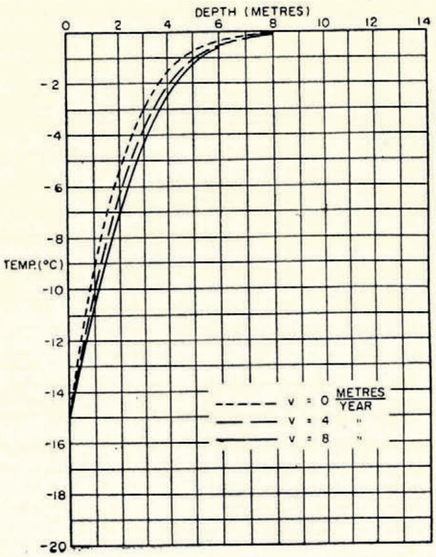
The initial temperature of the snow field has been taken to be 0° C. throughout. In other words, the effect of any earlier surface temperature fluctuations has been considered to be erased during the course of the previous summer, when the whole snow field is supposed to have risen to 0° C., as has been found to occur [3] for example in the *névé* region of the Aletsch Glacier. Therefore, referring to Fig. 1, when time $t=0$ the whole snow field is imagined to be at 0° C. throughout its whole volume. Three crosses on Fig. 1 (at $t=4$, 7 and 10 months) mark the times for which sub-surface temperatures have been calculated.

In these calculations a constant value of the diffusivity of the snow, $K=7 \cdot 3 \times 10^{-3}$ cm.^{2}/sec., has been used. For this value of K, the temperature wave in a non-accumulating snow field would travel downward at the rate of 17 meters/year. In actual conditions in a snow field K will not be constant, but, rather, may be expected on the average to increase somewhat irregularly with depth; a possible means of allowing for a certain variation of K with depth has been discussed earlier.[6] The velocity of accumulation, v, has also been assumed to be constant. In principle the method of calculating the sub-surface temperatures could be extended to cases where v is not constant, and even changes sign (that is, ablation occurs), but in practice such calculations would involve much labor.

In order to illustrate the blanketing effect of accumulating snow we have accordingly chosen to compute sub-surface temperatures for constant velocities of accumulation at the rates of $v=0$, 4 and 8 meters/year, for the times $t=4$, 7 and 10 months after the maximum temperature of the previous annual cycle occurred at the surface. Hence if, for example (see Fig. 1), the maximum







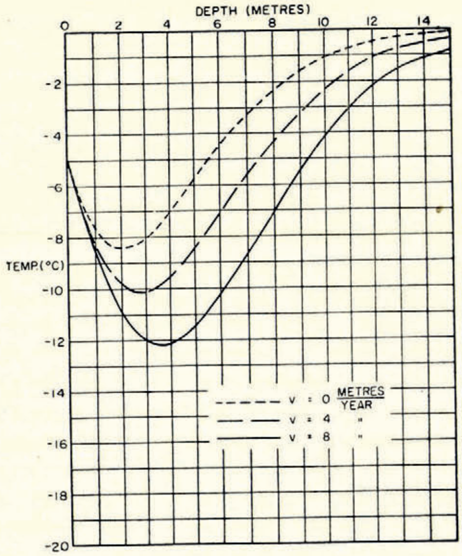


Fig. 1 (top left). The assumed temperature T of the surface of the snow plotted against time for one year. The crosses, at t=4, 7 and 10 months after maximum surface temperature, mark the times for which sub-surface temperatures have been calculated, as shown in the succeeding figures

Fig. 2 (bottom left). The sub-surface temperatures for t=4 months, for velocities of accumulation at the rate of v=0, 4 and 8 meters/year

Fig. 3 (top right). The sub-surface temperatures for t=7 months, for velocities of accumulation at the rate of v=0, 4 and 8 meters/year

Fig. 4 (bottom right). The sub-surface temperatures for t=10 months, for velocities of accumulation at the rate of v=0, 4 and 8 meters/year.

TABLE I

Depth x (meters)	Temperature (° C.)		
	v=0 meters/year	v=4 meters/year	v=8 meters/year
0	−15·0	−15·0	−15·0
1	− 9·2	−10·0	−10·6
2	− 5·4	− 6·2	− 7·0
3	− 3·0	− 3·8	− 4·4
4	− 1·6	− 2·1	− 2·5
5	− 0·8	− 1·1	− 1·2
6	− 0·4	− 0·5	− 0·5
7	− 0·2	− 0·3	− 0·2
8	− 0·1	− 0·1	0·0

Calculated values of sub-surface temperatures for t=4 months for three different velocities of accumulation, v. Fig. 2 (p. 252) is constructed from this table.

TABLE II

Depth x (meters)	Temperature (° C.)		
	v=0 meters/year	v=4 meters/year	v=8 meters/year
0	−18·7	−18·7	−18·7
1	−15·4	−16·7	−17·8
2	−11·8	−13·8	−15·7
3	− 8·6	−10·9	−13·2
4	− 6·0	− 8·1	−10·5
5	− 3·9	− 5·8	− 7·8
6	− 2·5	− 3·9	− 5·6
7	− 1·5	− 2·6	− 3·9
8	− 0·9	− 1·7	− 2·4
9	− 0·5	− 1·0	− 1·5
10	− 0·3	− 0·6	− 1·0
11	− 0·2	− 0·3	− 0·4

Calculated values of sub-surface temperatures for t=7 months for three different velocities of accumulation, v. Fig. 3 (p. 252) is constructed from this table.

TABLE III

Depth x (meters)	Temperature (° C.)		
	v=0 meters/year	v=4 meters/year	v=8 meters/year
0	−5·0	− 5·0	− 5·0
1	−7·6	− 8·3	− 8·5
2	−8·5	− 9·8	−10·8
3	−8·1	−10·1	−12·0
4	−7·0	− 9·6	−12·1
5	−5·7	− 8·4	−11·4
6	−4·4	− 7·0	−10·2
7	−3·3	− 5·6	− 8·6
8	−2·3	− 4·3	− 7·0
9	−1·6	− 3·2	− 5·4
10	−1·1	− 2·3	− 4·1
11	−0·7	− 1·6	− 3·0
12	−0·5	− 1·1	− 2·1
13	−0·3	− 0·8	− 1·5
14	−0·2	− 0·5	− 1·1
15	−0·1	− 0·4	− 0·8

Calculated values of sub-surface temperatures for t=10 months for three different velocities of accumulation, v. Fig. 4 (p. 252) is constructed from this table.

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surface temperature occurs in early August, times $t=4$, 7 and 10 months correspond to early December, March and June respectively.

The sub-surface temperature distributions for times $t=4$, 7 and 10 months are shown in Figs. 2, 3 and 4 respectively. They have been drawn from the computed temperatures shown in Tables I, II and III.

In Tables I and II the temperatures for $v=0$ meters/year have been calculated from the exact expression (see reference 6). This requires a numerical integration in order to compute each temperature. In Table III the $v=0$ meters/year temperatures have been calculated by a more rapid, approximate method used by Sverdrup [2] and Hughes and Seligman.[3] This approximation, which neglects the effect of a transient term, is satisfactory for $t=10$ months (Table III); it would, however, introduce errors as large as 0·2° C. in Table II and 0·4° C. in Table I, in their $v=0$ columns.

The temperatures in Tables I, II and III corresponding to velocities $v=4$ and 8 meters/year have been calculated by another approximation (see reference 6, p. 144), which may introduce individual errors of as much as about one-third or possibly even one-half a degree Celsius when $v=8$ meters/year, and correspondingly less error when $v=4$ meters/year. No doubt this accounts for the temperatures in Table I under $v=8$ meters/year at depths of 6 meters and deeper, being somewhat too high compared with those for $v=4$ meters/year (for example, for the depth 15 meters, $t=10$ months and $v=8$ meters/year, the approximate method of calculation gives a temperature 0·32° C. higher than the exact method). However, since the exact method requires considerably more labor, the approximation has been used.

Results and Conclusions

A study of Figs. 2, 3 and 4 shows that (1) accumulation at the rate of $v=8$ meters/year has a greater effect in decreasing the sub-surface temperature than when $v=4$ meters/year, as would naturally be expected, and (2) that the difference in sub-surface temperatures, though relatively small when $t=4$ months (Fig. 2) is quite noticeable and significant for $t=7$ and 10 months (Figs. 3 and 4), particularly for the latter case.

After a winter of large and steadily prolonged accumulation amounting to, say, 4 to 6 meters of snow or more, the buried cold wave should, therefore, be of considerably greater proportions, other things being equal, than after a winter of little accumulation. This larger cold wave should introduce a noticeable increase in the effect of the subsequent freezing of melt water and of ice band formation in the summer months that follow.

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