*Star Clusters: From the Milky Way to the Early Universe*
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# Phase space complexity of star clusters: Fresh observables for old and new questions

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**Abstract.** The blooming era of precision astrometry for Galactic studies truly brings the rich internal dynamics of globular clusters to the centre stage. But several aspects of our current understanding of fundamental collisional stellar dynamics cannot match such new-generation data and the theoretical ambitions they trigger. This rapidly evolving context offers the stimulus to address a number of old and new questions concerning the phase space properties of this class of stellar systems.

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## 1. A new observational and computational landscape

Globular clusters are among the first stellar structures which have emerged at the dawn of the formation of galaxies. They play many roles throughout cosmic time, from survivors of intense star formation epochs in the early Universe, via galactic beacons carrying crucial information about the assembly history of their host environment, to rich cradles of stellar-mass black holes as prime candidate sources of gravitational waves.

In April 2018, the second Data Release of the European Space Agency mission Gaia has opened the era of 'precision astrometry' for the study of the Milky Way (Gaia Collaboration 2018). This new generation of astrometric data, complemented by proper motion measurements from decades-long campaigns with Hubble Space Telescope (e.g., see Bellini, Libralato, Watkins, Milone, this volume) and state-of-the-art spectroscopic surveys (e.g., see Ferraro *et al.* 2018; Kamann *et al.* 2018, also in this volume), enable us to unlock, for the first time, the full 'phase space' of globular clusters in the six-dimensional position and velocity space.

Such information, coupled with a detailed chemical characterisation of individual member stars, opens a new 'golden age' for the study of the internal dynamics and stellar populations of Galactic globulars. Nonetheless, to fulfil the promise of 'precision astrometry', a careful understanding of many systematic effects is crucial, especially in crowded fields at the centre of dense clusters (e.g., see Pancino *et al.* 2017 and Bianchini, Vasiliev, this volume).

Another transformational shift is unfolding on the computational side: on the one hand, thanks to cost-effective accelerator hardware (GPUs) and recent software developments (Nitadori & Aarseth 2012), the gravitational million-body problem has finally been 'cracked' and the exploration of the entire dynamical evolution of these systems

- on a star-by-star basis - is within reach (see Heggie 2014; Wang *et al.* 2016). On the other hand, the study of fully resolved star clusters is still beyond the capabilities of current cosmological simulations, but the numerical resolution is steadily increasing and tantalising connections between globular cluster systems and the properties of their host galaxies are already emerging (e.g., see Renaud, Li, Reina-Campos, Grudić, this volume).

Lastly, with the discovery of an enigmatic population of satellites (e.g., see Koposov *et al.* 2015) lying in the transition region between dwarf spheroidal galaxies and classical globulars in size-luminosity space, the spectrum of low-mass stellar systems in the Milky Way halo is becoming progressively richer, triggering fundamental questions about the distinction between 'dwarf galaxies' and 'star clusters' (see also Jablonka, Ishigaki, Yong, this volume).

This rapidly evolving context offers the stimulus and the opportunity to address a number of old and new questions in stellar dynamics concerning the phase space properties of this class of stellar systems, as summarized below.

## 2. What are the stability properties of rotating, anisotropic spheroidal equilibria?

Recent kinematic studies based on Gaia Data Release 2 have further confirmed the growing evidence that the presence of internal rotation in globular clusters is much more common than previously assumed (e.g., see Bianchini *et al.* 2018; Sollima *et al.* 2019, also in this volume). Yet the stability properties of spheroidal rotating stellar systems are largely unexplored and the connection with the corresponding classical theory of rotating fluids (e.g., see Chandrasekhar 1969) is only partially understood.

We have taken a first step in this direction by considering the behaviour of spherically symmetric systems with differential rotation and anisotropy in the velocity space. Specifically, we have investigated the stability of several sequences of Plummer spheres, in which the total angular momentum, as well as the degree and flavour of anisotropy (as first proposed by Dejonghe 1987) in the velocity space are varied via the 'Lynden-Bell demon' (Lynden-Bell 1962)†. We have collaborated in a comparative analysis between linear theory and N-body methods, applied to the same initial equilibria. In the first case, the 'response matrix method' (see Kalnajs 1977 and subsequent applications) is tailored to handle spherically symmetric, rotating configurations. The shapes, pattern speeds and growth rates of the systems' unstable modes have been explicitly computed. We have charted a marginal stability boundary in the parameter space defined by the degree and flavour of velocity anisotropy and the total angular momentum. When rotation is introduced, two sequences of growing modes are identified corresponding to radially and tangentially biased anisotropic spheres, respectively. We believe that the instabilities identified in tangentially anisotropic spheres are novel. For radially anisotropic spheres, growing modes occur on two intersecting surfaces (in the parameter space of anisotropy and rotation), which correspond to 'fast' and 'slow' modes, depending on the net rotation rate. We have also formulated some generalized, approximate stability criteria which extend the classic 'radial orbit instability' condition (see Polyachenko & Shukhman 1981) to a broader domain. Full details are offered in Rozier *et al.* (2019); such a study is also presented as an extended summary included in this volume. Further investigations largely based on N-body simulations are currently in progress (Breen *et al.* 2020b).

† A Python script which can be used to generate such initial conditions is publicly available at https://github.com/pgbreen/PlummerPlus.

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## 3. What are the implications of 'kinematic complexity' on the long-term evolution of collisional systems?

Similarly, the study of the structure and dynamical evolution of collisional stellar systems has been traditionally pursued under a relatively stringent set of simplifying assumptions, such as isotropy in the velocity space and the absence of ordered motions, which limit significantly the opportunity to explore the kinematic richness of observed globular clusters. To develop a fundamental understanding of the implications of such physical ingredients, we investigated the dynamical evolution of isolated equal-mass star cluster models by means of direct N-body simulations (performed with NBODY6, see Nitadori & Aarseth 2012), focusing on the effects of the presence of primordial anisotropy and rotation in the velocity space.

First, we found evidence of the existence of a monotonic relationship between the moment of core collapse and the amount and flavour of anisotropy in the stellar system. Specifically, equilibria characterized by the same initial structural properties (Plummer density profile) and with different degrees of tangentially biased (radially biased) anisotropy, reach core collapse earlier (later) than isotropic models. We interpreted this result in light of an accelerated (delayed) phase of the early evolution of collisional stellar systems, which we have characterized both in terms of the evolution of the velocity moments and of a fluid model of two-body relaxation. For the case of the most tangentially anisotropic model ('Einstein spheres'), the initial phase of evolution involves a catastrophic collapse of the inner part of the system, which continues until an isotropic velocity distribution is reached.

Second, we have confirmed that, in the presence of internal rotation (as introduced via the 'Lynden-Bell demon', see previous Section), the systems reach core collapse more rapidly compared to the corresponding non-rotating ones. This result is not surprising, as several studies based on Fokker-Planck (e.g., see Einsel & Spurzem 1999) and N-body methods (e.g., see Ernst *et al.* 2008) had already identified such a behaviour. But, in this context, we wish to stress that a comparison between the core collapse times of rotating and non-rotating systems can be truly meaningful only if the structural properties of the initial equilibria are proved to be the same. The 'acceleration' of the dynamical evolution noticed in rotating systems has been traditionally ascribed to the effects of the so-called 'gravogyro instability' (first introduced by Inagaki & Hachisu 1978 for idealised fluid systems, and later applied to the interpretation of the beahviour of stellar systems by Akiyama & Sugimoto 1989). In this context, we have showed that the characteristic 'spin up' of the core in the presence of outward angular momentum transport (often interpreted as the signature manifestation of the 'gravogyro instability') is actually present also in initially non-rotating models. Finally, we have identified a general scaling of the time of core collapse with the relaxation time of the core of the systems, both in the case of anisotropic and rotating configurations. Core collapse is indeed a relaxation-driven phenomenon, and so it is somehow natural to expect that the time of core collapse is determined by the time of relaxation of the central regions. Full details are presented in Breen *et al.* 2017, 2020a. This study represents a first step towards a comprehensive investigation of the role played by 'phase space richness' in the long-term dynamical evolution of star clusters and offers an essential conceptual base to develop a deeper understanding of the possible nexus between kinematic complexity and the presence of black holes of different classes of mass in collisional systems.

## 4. What is the degree of 'phase space hysteresis' of collisional systems?

The previous two Sections have provided a summary of some recent progress towards a more complete understanding of the role of angular momentum and velocity anisotropy

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in the dynamical evolution of collisional stellar systems. Such a foundation now enables us to attack the more complex case of (equal-mass) systems characterized by two different components, with the goal of assessing the ability of a collisional system to 'keep memory' of its initial conditions in phase space. This question is particularly relevant for the dynamical dimension of the multiple population phenomenon.

In fact, a very promising but hitherto largely unexplored angle concerns the phase space characterisation of the different stellar populations in globular clusters. There is clear evidence that 'second population' stars (i.e. those enhanced in Na and depleted in O, relative to halo stars) are more centrally-concentrated than 'first population' stars (e.g., see Lardo *et al.* 2011). The recent kinematic studies mentioned in the previous Sections imply that Galactic globulars likely have had some rotation when they were formed (e.g., see Bekki 2010), leading to differences in the distribution of angular momentum between the two populations (e.g., see Hénault-Brunet*et al.* 2015. A recent investigation has substantiated this expectation (Cordero *et al.* 2017) and there have also been reports of different degrees of rotation and velocity anisotropy for distinct populations based on HST and Gaia DR2 proper motions (see Bellini *et al.* 2015; Libralato *et al.* 2019; Cordoni *et al.* 2019, also included in this volume).

The development of theoretical models capable of coping with this new level of chemical and dynamical complexity is therefore required in order to formulate a sound physical description of this phenomenon and to provide meaningful dynamical constraints on the numerous multiple population formation scenarios. For these reasons, we have conducted a numerical investigation aimed at understanding the fundamental aspects of the long-term evolution of the internal kinematics of multiple stellar populations in globular clusters. Our N-body models enable us to study the cooperative effects of internal, relaxation-driven processes and external, tidally-induced perturbations on the structural and kinematic properties of multiple-population globular clusters. To analyse the fundamental dynamical behaviour of the multiple stellar populations in a variety of spin-orbit coupling conditions, we have considered three reference cases in which the tidally perturbed star cluster rotates along an axis oriented in different directions with respect to the orbital angular momentum vector. We focussed specifically on the characterisation of the evolution of the degree of differential rotation and anisotropy in the velocity space, and we quantified the process of spatial and kinematic mixing of the two populations. In light of recent and forthcoming explorations of the internal kinematics of this class of stellar systems by means of line-of sight and astrometric measurements, we also investigated the implications of projection effects and spatial distribution of the stars adopted as tracers. The kinematic and structural richness emerging from this study further emphasises the need and the importance of observational studies aimed at building a complete kinematic picture of the multiple population phenomenon. Full details are presented in Tiongco *et al.* 2019; an extended summary is available also in this volume.

## 5. How should we approach the regime at the interface between collisional and collisionless dynamics?

The attention to the possible degree of 'hysteresis' possessed by collisional stellar systems (see previous Section) is ultimately motivated by the desire the infer some clues to the origin of such systems. In fact, the formation of globular clusters in a cosmological context remains a major challenge for astronomical research, and it has an inevitably tangled connection to the multiple population phenomenon. But the opportunity to characterise the properties of 'proto' globular clusters in the early Universe will soon be within reach, thanks to large-scale numerical simulations with progressively higher resolution (e.g., see Li *et al.* 2017) and forthcoming observational facilities with greatly improved sensitivity and wavelength coverage, in particular the James Webb Space Telescope (Renzini 2017). Actually, such an opportunity has already started to materialize: a number of studies of compact star-forming objects identified as magnified sources at intermediate to high redshifts have been conducted (see Vanzella 2017, which is also included in this volume, and Bouwens *et al.* 2017; Kikuchihara *et al.* 2019). Such objects appear to lie at the interface between 'proto' galaxies and 'proto' star clusters and their distinction is often non-trivial (an unbiased search for stellar systems in this regime in a cosmological simulation has been conducted by Phipps; a summary is included this volume).

Also for local Universe studies the nature of faint stellar systems at the interface between classical globular clusters and dwarf galaxies is currently at the centre of much attention (e.g., see Koposov *et al.* 2015), as they may represent the lower limit to the dark matter clustering scale and their properties and demographics provide crucial insight into many open problems in galaxy formation. Dynamical investigations of such objects are usually polarised around a sharp dichotomy: if they are approached as star clusters, they are studied as collisional, dark matter-free objects; if they are treated as satellite galaxies, they are considered as collisionless and dark matter-dominated. But many stellar systems actually fall into the regime at the interface between collisional and collisionless dynamics, and their dark matter content is hard to pin down (e.g., see Laevens *et al.* 2015).

Driven by these motivations, we have approached the study of the equilibrium and evolutionary properties of collisional stellar systems in a small dark matter halo, with a number of interesting results (see Peñarrubia *et al.* 2017). So far only a handful of studies has focused on this topic (e.g., see Mashchenko & Sills 2005) and the 'gravothermodynamics' of collisional systems in the presence of a dark matter potential is largely untouched. This investigation has the potential to enrich our fundamental understanding of collisional stellar dynamics in a domain so far unexplored. Such an effort may offer us new insights into the perplexing regime at the interface with collisionless dynamics and a possible framework for the dynamical interpretation of the 'hybrid' stellar systems which are currently identified in both the local and the early Universe.

## 6. Conclusions

The traditional picture of globular clusters as fully relaxed, isotropic, non-rotating, spherical systems characterised by a single very old stellar population cannot cope with the complexities emerging from the new-generation data which are now available and the theoretical ambitions that they stimulate. The formulation of a more realistic dynamical interpretation of this class of stellar systems is therefore pivotal to exploit the synergy between precision astrometry enabled by Gaia and the Hubble Space Telescope, the forthcoming massive photometric and spectroscopic surveys of the Milky Way, and the exploration of the first sources of light in the early Universe promised by the James Webb Space Telescope and the Extremely Large Telescopes. This contribution briefly summarizes our current efforts to address four old and new questions concerning our fundamental understanding of the internal dynamics of globular clusters, mostly because we believe that 'even castles in the sky can do with a fresh coat of paint' (Murakami 1992).

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