ON THE VELOCITIES ADDITION RELATIVISTIC EQUATION

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ABSTRACT

Beginning with an analysis of the relativistic equation for the addition of velocities, whose circularity has been mathematically demonstrated, the current status of the theory of special relativity is examined. The article contains an updated overview of the theoretical research and recent experimental evidence. They show that special relativity still remains, for its theoretical power and experimental confirmations, Einstein's original program, supplemented by Minkowski's reformulation. Chapters I-II highlight the experimental, not logical-deductive, foundations of special relativity and the consequent domain of applicability. Chapter III addresses the issues open on the level of theory and mathematical formalism (the inertial system as idea-boundary, paradoxes, authentic and not, the correspondence between the values of space-temporal interval and proper time). In chapter IV the conclusions: the equation of addition of velocities as a mere tautology; the observational basis, not logico-deductive, of the invariance and velocity-boundary property of speed of light, c; the absence of confirmed established experiments (indirect confirmations of dubious interpretation) for length contraction, as opposed to time dilation; the incompatibility between relativity and quantum theory attested to by entanglement; the unresolved problems of the 4-dimensional formalism, i.e., the theory of spacetime. Special relativity represents, in short, an effective theory with verified experimental confirmation, but nevertheless lacks sufficient elements to assume it as a complete and definitive description of the spacetime relationships.

Chapter I : THE FORMULA FOR ADDING VELOCITIES IN SPECIAL RELATIVITY

The analysis of the relativistic equation of addition of velocities is carried out here as the starting point of a broader review of the present state of Special Relativity (SR), i.e. the theory of flat spacetime. The article is primarily based on kinematics and dynamics, which form the basis from which the main results of the theory can be derived, deferring detailed analysis of the other aspects, such as electromagnetic theory and electrodynamics, to a second step. Why start with the addition of velocities? This is a primary point in the history of physics, which affects the theory of light and represents a decisive junction in mechanics, both classical and relativistic, and in general in the study

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of the spacetime assumptions of physics. The history of which is, in fact, a significant part of the history of what light is. Einstein derived the equation (like the dilation of time and the contraction of distances, the Maxwell-Hertz transformations, the aberration theory, and other remarkable results) in his 1905 memoir, published in "Annalen der Physik", *Zur Elektrodynamik bewegter Körper*, from two postulates: that of relativity, with the associated concept of an Inertial Reference System(IRS), and that of the law of propagation of light in vacuum. He also based his work on the Lorentz transformations, which formalise, according to the form of Maxwell's[1] laws, the rules for describing variations in time and space measurements in linear coordinate transformations between inertial systems, and which form the mathematical core of SR. The relativistic formula (or theorem, according to the name sometimes used by Einstein) for the addition of velocities is:

$$V = \frac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}} \qquad (1)$$

It introduces, compared to the classical formula, ($V= v_1 + v_2$), the denominator $1+\frac{v_1}{c}\frac{v_2}{c}$, i.e. the simple sum of the velocities of the IRS is replaced by the ratio between this sum and a quantity obtained by adding to unity the product of the quotients between the velocities of the IRS considered and the speed of light in vacuum *c*. The correction is given by the denominator. The new formula allows the set of experimental results to be framed in the complete range of velocities from

$$\text{a)}\ \frac{u}{c}= 0 \quad \textit{to} \quad \text{b)}\ \frac{u}{c}= 1 \qquad (2)$$

The relativistic equations are reduced to the classical equations when $\frac{\text{v}}{c}$ is very small,

$$\frac{u}{c} \ll 1 \qquad (3)$$

approximately when the Lorentz factor is below 0.8 *c*. It goes without saying that the law of addition of velocities, codified in classical mechanics, implied that the speed of light had to take on different values for two observers in relative motion to each other, and was therefore at odds with the invariance of the speed of light resulting from Maxwell's equations. The classical formulation of the principle of relativity does not agree with electromagnetic interaction. Light does not obey traditional kinematics. Both theoretically and experimentally, the formula for the addition of velocities was proving to be a weak point in the Galilean-Newtonian vision. With the contribution mainly of Maxwell, a major problem had thus opened up. After the work of W. Voigt[2], J. Larmor,[3] H. A. Lorentz,[4] G. F. FitzGerald,[5] before A. Einstein[6] and H. Poincaré[7] himself had already derived the transformation equations. The fundamental equations had, therefore, already been written before, but without arriving at the general character of his reasoning on relativity. Maxwell's theory was invariant with respect to a new and different transformation system, the Lorentz system. In other words, to preserve the form of Maxwell's equations, the sum of velocities cannot consist of the simple vector sum.

In 1905, Einstein put things into a complete picture. There were two possible ways: a) electromagnetic phenomena are describable in a privileged inertial system (a particular IRS bound to the ether, in virtue of which absolute motion and absolute stillness would make sense) and, therefore, the electromagnetic theory is incorrect; 2) the principle of relativity is also valid for electromagnetic phenomena, i.e. the Galilean transformations are incorrect and, therefore, classical kinematics must be modified. Relativity, historically, is developed to preserve the laws of electromagnetism, but this entails re-examining fundamental concepts of physics. With SR, the Galilean principle of relativity is generalised and Newtonian simultaneity is abandoned. SR is, however, a theory of flat spacetime, as it is 'restricted' to IRSs, while general relativity (GR) will be a theory concerning arbitrary reference frame (*RF*). After Einstein's formidable shake-up, it was gradually accepted that, having recognised Maxwell's laws as correct, mechanics had to be modified. Thus the principle of relativity took on a general meaning: the laws of physics, not just those of mechanics, are the same in all inertial systems. Even the ether, after tenacious resistance (recall Lorentz and Poincaré, who were always convinced of its existence), was finally set aside. For Einstein, the state of stillness with respect to the ether was equivalent to an untenable and superfluous state of absolute stillness. Decisive were the null attempts to experimentally detect motion with respect to the ether. In summary, Maxwell's equations do not have a symmetrical structure with respect to Galileo's transformations, they are instead symmetrical with respect to the Lorentz transformations.

In classical physics, temporal and spatial coordinates are mutually independent. In SR they are interdependent; in fact:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\beta^2}} \qquad (4)$$

as well as

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad (5)$$

in the former the spatial component appears, in the latter the temporal component.

Having made this premise, let us take the relativistic equation of addition of velocities and do some calculations with it, very elementary and contained to the essentials, which lead to an interesting result and, perhaps, even of a certain originality.

(A)

The first calculation says that if $v_1 = c$ or equivalently $v_2 = c$, the result will always be c, regardless of the value of v_2 or of v_1. Let us assume $v_1 = c$. We will have:

$$V = \frac{c+v_2}{1+\frac{c\,v_2}{c^2}} = c \;; V = \frac{c+v_2}{\frac{c^2+cv_2}{c^2}} = c \qquad (6)$$

With identical result, assuming $v_2 = c$.

This is interpreted as:

a) c in vacuum does not compose with any other velocity ;

b) c is the maximum possible speed.

Two results proven by experiments, but not derivable from the formula, which reveals a circular character.

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(B)

Let us now imagine, as a second calculation, with a Gedanken - Experiment, a world without light, a dark universe. Let us then take, mathematically instead of c, the speed of sound S = 331 m/sec.

In this, which we will call “Moles world”, using Einstein's formula with $v_1 = S$, we will have:

$$V=\frac{S+v_2}{1+\frac{Sv_2}{S^2}}=\frac{S^3+S^2v_2}{S^2+S\,v_2}=\frac{S(S^2+S\,v_2)}{S^2+S\,v_2}=S \qquad (7)$$

In the “Moles world”, the speed of sound does not compose with the others. Moreover, even in our world, it is independent, unlike frequency, of the motion of the source (in fact $v=\frac{\lambda}{T}$ but $f=\frac{1}{T}$, therefore $v=\lambda f$), although, as is well known, it is not a constant. Moles might plausibly think, in the absence of light, that the speed of sound is the maximum possible speed.

(C)

Let us apply, as a third calculation, the relativistic addition formula in the situation of one of Einstein's famous railway examples: on a train, running at a speed of 50 km/h, a passenger walks in the same direction at a speed of 10 km/h. In this case, the universe considered will be an imaginary universe-train, on which passengers, messengers, flies, ants etc. move. In the denominator we will, of course, put not the speed of light, but the speed of the train. In this case we will have

$$V=\frac{50+10}{1+\frac{50\times 10}{50^2}}=\frac{60\times 2500}{2500+500}=50 \qquad (8)$$

In the train-universe, applying the adapted formula, the speed of the train does not compound with the other speeds and the inhabitants of that universe, devoid of electromagnetic waves and sound waves, could plausibly think, comforted by the formula, that it constitutes the maximum possible speed.

These very simple calculations lead to the following question: is the non-additivity of the speed of light c a derivation in the logical-mathematical formalism or is it instead the result of experimental observations? The mathematical formalism considered shows an obvious circular character. This is made even more explicit by the following generalisation :

$$\text{a)}\quad V=\frac{A+B}{1+\frac{A\times B}{A^2}}=A\ ; \quad \text{as well as} \quad \text{b)}\quad V=\frac{A+B}{1+\frac{A\times B}{B^2}}=B \qquad (9)$$

That is, if we substitute any two letters in the numerator and the same letter in the denominator for any two letters in the formula, the result will always be the number entered. The formula describes this relationship and clearly cannot prove anything about the non-composability of the speed of light c. Even the very astute Yuan Zhong Zhang, in the *Special relativity and its experimental foundations*, writes: «However, Einstein's law of the addition of velocities gives a different result»[8].

In reality, as we are demonstrating, Einstein's equation gives nothing more than a tautology. But it is clearly not a question of one equation or the other being stronger or

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more consistent. Decisive in overcoming Newtonian mechanics was another factor: the electrodynamics of Faraday and Maxwell, as Einstein explains very clearly.[9]

Even if we wanted to obtain more general results, for example by relating, with the Lorentz transformations and the equations for the direct and inverse transformation of velocities, the velocity v of a body with respect to an observer in the IRS S with the velocity of the same body seen by an observer in the IRS S', moving at velocity U with respect to S, the form of the relations for the sum of the velocities would not change. If the observed event, seen from S' were a ray of light along the x-axis, it is well established that the measured velocity, v′, would be equal to c, and the velocity, seen from any other IRS, would always be equal to c and, moreover, would not be addable with any other velocity. There is no need to write down the equations, as the analysis conducted earlier is also applicable in this case.

Returning to the formula in question, it has a multitude of applications. If we add two velocities less than c, the result is always less than c. If we use it in the context of the velocities we have to deal with in our ordinary, low-speed context, we can verify maybe the greater precision of the relativistic formula compared to the classical one, but with insignificant and ultimately negligible differences. If, finally, either velocity is greater than c, even by a small amount, the result will be greater than c. If we add, e.g. $\frac{5}{4}\,c$ with $\frac{1}{3}\,c$ or with $\frac{3}{5}\,c$, applying the relativistic formula will always result: $V > c$. What does this mean? One can legitimately conclude that the "demonstration" proves nothing. The only way to 'prove' the non-composability of c is to devise a different formalism or, as has historically happened, to rely on the experimental route. It must of course be borne in mind that if, in Lorentz transformations, we place $v > c$, then $\sqrt[2]{1 - v^2/c^2}$ becomes imaginary, which makes no sense, as the coordinates must be real.

Let us leave aside, for the moment, the discussion of "the one-way speed of light" and "the two-way speed of light", pointing out that the references in the text to the experimental verification of the invariance of c are to be understood, unless otherwise stated, as referring to "the two-way speed of light", which is the only method that does not presuppose any clock synchronisation procedure, because in the round-trip only one clock is needed, at the point where the closed trajectory begins and ends.

Let us also leave out, to return to it in the last two Chapters the question of the relationship between SR and GR. The spacetime of GR is very different from the spacetime of SR, in which mass-energy has zero value, and from the *definitions* or *assumptions* and/or *conventions* on which, as we shall see, it is based.

Special or restricted relativity has a long theoretical and experimental history. Before, during and after Einstein's creative breakthrough, soon complemented by Minkowski's[10] mathematical reformulation, there was a vast theoretical and experimental production. We are confronted with an immense literature. The production and discussion is still going on, also because, due to the relationship between quantum gravity and SR, as the former implies the possibility of SR violations at very high energy scales and ultra-small distances, and, furthermore, because many believe that any violation of the CPT theorem (charge, parity, time symmetries) would imply a violation of Lorentz invariance, a veritable renaissance of SR studies has occurred. Thus, starting with the aforementioned contributions of Voigt, Larmor, FitzGerald, and the fundamental works of Maxwell, Lorentz and Poincaré, the theoretical elaboration contemporary and subsequent to

Einstein has focused on issues of great importance: the problem of the ether and the electrodynamics of moving bodies, relative motion, the relativity of space and time, the invariance of the speed of light in vacuum, and the theory of spacetime as the basic four-dimensional structure of the universe. There is no shortage of recent developments, some of which are in an explicitly broader thematic field. See, as an example, DSR, i.e. *Doubly Special Relativity* (G.Amelino-Camelia ,[11] T. G. Pavlopoulos ,[12] J. Magueijo ,[13] L. Smolin[14]), which introduces, with a truly interesting point of view, in addition to the maximum speed constituted by light, a maximum energy scale and a minimum length scale, independent of the observer. Among the most stimulating recent analyses of relativity, both narrow and general, is certainly that of DiSalle,[15] which clearly focuses on the transformations that have taken place in the physics of space and time.

For a study of the historical development of SR, on an experimental level, the most widely used references for this work are a) T. Roberts and S. Schleif, *What is the experimental basis of SR*?[16] which offers an index close to completeness; b) Yuan Zhang Zhong, *"Special Relativity and its experimental foundations"*, focusing particularly on the Test Theories of SR;[17] c) *'Modern tests of Lorentz Invariance'* on Living Reviews.[18] In addition, D. Mattingly,[19] *Modern tests of Lorentz Invariance*, and M. W. Clifford,[20] which mainly focuses on experimental tests on GR. The development of observations and experiments, in the field of SR, runs through the various chronological phases and the multiplicity of aspects: from the experiments of the pre-1905 period and years immediately following, including the experiments of the 'heroic' phase (Römer,[21] Arago,[22] Fresnel,[23] Fizeau,[24] Sagnac,[25]...), tests on the isotropy of the round-trip speed of light, maser tests, tests on the speed of light one-way or from moving sources, up to tests on the principle of relativity and Lorentz invariance, on the isotropy of space, on time dilation and contraction of lengths, on the Doppler effect, to relativistic kinematics tests, Test Theories and further modern tests on CPT and Lorentz invariance, on *Cosmic Background Radiation* (CMBR), on the constancy of physical constants and so on. It is not the task of this article to describe and even less to discuss such a large number of experiments, and we would not have the necessary expertise for many of them, but what we do have to do is to understand the existing literature and assume the results that have emerged, when they are generally accepted by the physics community.

As Roberts and Schleif,[26]: «SR is not a mathematical game or just a hypothesis. SR is a physical theory that has been well tested many times». Again: « ... as of this writing there are no reproducible and generally accepted experiments that are inconsistent with SR, within its domain of applicability». SR is a genuine scientific theory, it makes powerful predictions, which are therefore fallible and, up to the time of writing, not experimentally contradicted. In short, this is the conclusion drawn and stated by the vast majority of researchers.

Among the experiments that are generally considered fundamental, not least because of the influence they have historically exerted, are certainly:

a) the Michelson (1881) and Michelson + Morley (from 1887, subsequently repeated and refined by them and others) experiments on the isotropy of light and direction dependence;
b) the Kennedy-Thorndike (1932) experiment on the velocity dependence of the observer;

c) the Ives-Stilwell experiment (1938) on the transverse Doppler effect and consequently on time dilation.

To these experiments, the importance of which, beyond the historical perspective, is beyond question, one can add many others, depending on one's theoretical and experimental views and preferences, such as the Trouton-Noble experiment (1902-3), one of the most brilliant on the subject of 'motion through the ether', and the electrodynamic equivalent of the Michelson-Morley optical experiment, or the Hafele-Keating experiment (1972) on clock comparison and the Twin Paradox. The field is vast, especially if one includes astrophysical tests, also using cosmological sources, and experiments carried out with lasers, masers, optical resonators, high-precision atomic clocks and in general with the numerous modern equipment and new experimental approaches.

Special attention must be paid to two chapters, mentioned above, in the experimental history of SR : the *Test Theories of SR*[27] and the *Tests of CPT and LI (Lorentz Invariance)*.[28] Under stress is the Lorentz covariance, i.e. the invariance of all physical laws for inertial reference systems.

The contribution of Test Theories of SR to the analysis of special relativity and, in particular, its experimental foundations must be considered extremely valuable. In short, their common character consists, under different assumptions (preferred reference, LI violations, one-way or two-way speed of light), in generalising Lorentz transformations by introducing additional parameters. They, in essence, provide a mathematical framework for analysing the results of SR experiments and can predict results that are different or not from SR predictions. If the values of the "fitted" parameters differed from SR predictions, the experiments would not conform to SR. Four test theories are best known: 1) H. P. Robertson (1949);[29] 2) W. F. Edwards (1963);[30] 3) R. Mansouri and R.U. Sexl (1977) ;[31] 4) Y. Zhong Zhang (1995).[32] A different and broader approach, which also embraces the Standard and GR model, is the Standard-Model Extension (SME). As Zhang made clear, the Edwards transformations (who assumes, like Einstein, that the two-way speed of light is the constant *c*) are a generalisation of the Lorentz transformations, and the Mansouri-Sexl transformations are a generalisation of the Robertson transformations (whose postulate, more general in the sense that it "relaxes" that of Einstein - Edwards) assumes that the bi-directional velocity is equal to the one-directional velocity, in both directions, and is anisotropic and independent of the motion of the source. «So the MS transformation predicts the same observables effects as the Robertson transformation, just as the Edwards transformation does with the Lorentz transformation».[33] Robertson, and MS, predict different results from SR and Robertson's conclusion is that: "The kinematics...of physical space-time is thus found to be governed by the Minkowski metric, whose motions are the Lorentz transformation.[34] The Robertson (and MS) transformations constitute a test of "two-way speed of light", not "one-way speed of light", as the observable difference is only the anisotropy of the former. Zhang proposes a unified test theory, which embraces the other three and affirms the impossibility of test theories that are not reducible to those discussed. Zhang demonstrates,[35] among other things, that any theory based upon the existence of an aether is experimentally indistinguishable from SR and has an aether frame which is unobservable. Referring in particular to Robertson and Zhang for a detailed examination, we can limit ourselves here to what is closely related to the present work. Robertson's empirical results validate, based on the three experiments mentioned above, Lorentz transformations to an accuracy of ~0.1 %, while Zhang points out that modern experiments determine Lorentz transformations to within a few parts per million. Ultimately, while there is no shortage of those who criticise or reject SR for the most diverse reasons, including the alleged lack of empirical evidence, the vast majority of physicists and scientists in general agree that it has been experimentally verified and confirmed in many different ways.

The CPT theorem, first introduced by J. Schwinger[36] in 1951 and demonstrated a few years later by Lüders[37] and Pauli,[38] states the symmetry of physical laws with respect to charge (C), parity (P) and time (T) reversals. As is well known, violations of P symmetry (weak interaction), CP symmetry (neutral mesons), C symmetry and T symmetry emerged in the following years. Oscar Greenberg[39] proposed in 2002 a demonstration that violation of CPT would imply violation of Lorentz invariance. If Greenberg was right, we would be in the condition 'Simul stabunt, simul cadent'. Experimental research carried out recently has not come up with any direct evidence (V.A.Kostelecky and N.Russel).[40] These observations should lead to more systematic experimental verification of the CPT theorem and its interrelation with LI, before assuming violations of the latter, which appears much more solid and confirmed than the CPT theorem.

More generally, particularly in the last three decades, many experiments have been conducted and new experimental techniques introduced, both in the terrestrial and astrophysical domains (e.g. with LLR - Lunar Laser Ranging), tending to detect violations of Lorentz invariance or symmetry (on the speed of light, anisotropic differences, time dilation and length contraction, the principle of relativity and other SR predictions) and it can be said that, so far, no L.I.V. has been confirmed and ascertained (see in particular Mattingly,[41] Kostelecky V.A., Russel N., Liberati S.[42]). However, the experimental hunt for Lorentz violations continues and no one can rule out the detection and ascertainment of possible violations (e.g. M.Pospelov and M.Romalis).[43] We hope, anyhow, to obtain important results from experiment as Cherenkov Telescope Array and Hermes project, the inter-planetary Network.

Having made these brief remarks, necessary to emphasise that the examination of SR, at the present state of studies, must look at a broad spectrum of both experimental research and theoretical production, the point of view that guides the present work is that the core of SR still remains Einstein's original programme, supplemented by Minkowski's mathematical reformulation. This is due to its theoretical strength, results and strong experimental confirmation, with the exception of quantum entanglement. This view is in agreement with the fact that the correctness of SR in the Einstein-Minkowski formulation is generally assumed. This 'programme' also forms the basis of general relativity and the path that leads to quantum field theories.

One of the main lines of the paper concerns the discussion of the logical-deductive and observational-empirical foundations of SR. Apart from the historical dynamics, interesting though it may be, and, indeed, the relationship between theory and experience in Einstein's thought is analysed in Chapter II, what matters above all is the present status of the theory. Following the development of theoretical work and experimental evidence, it becomes quite clear that the foundations of SR are inductive and empirical, rather than logical-deductive. It will be seen that this also applies to Einstein. Even a scientist of Robertson's calibre underestimates this point when he claims that the three experiments of Michelson-Morley, Kennedy-Thorndike and Ives-Stilwell provide empirical evidence that the 3 parameters ($g_0\ g_1\ g_2$) can be taken as independent of the observer and thus replace «…so far as possible " Einstein's postulates of relativity, "...obtained deductively by Einstein… with "facts drawn from experience"».[44] The relationship between postulates and empirical data in Einstein will be examined in detail in Chapter II. One cannot fail to observe, however, that the restriction 'so far as possible' is rather dubious in a demonstrative procedure and that the replacement with empirical data is by no means straightforward. Einstein starts with two postulates, the principle of relativity and 'the postulated invariance of *c*', and proceeds with rods and clocks, with which to measure intervals of space and time. But it is the results of experience that decide the theory. SR is not based on deductions, but on the validation of theoretical ideas through observations. This is not the same as saying that the theory is entirely based on observations and entirely confirmed by observations. This aspect will be analysed in Chapter III. Notwithstanding, in fact, what is emphasised about the experimental confirmations of SR, one can, as will become clearer, observe and support: 1) the persistence of points that are not experimentally defined; 2) the presence of theoretical questions, far from marginal, that are still open.

There is one argument, which has methodological priority over the other aspects. Since speed is a relationship between a spatial distance and a temporal interval, any reasoning about speed presupposes the possibility, in principle, of measuring distances and durations. This is the theme of simultaneity and/or synchronisation of clocks. We will deal with this theme at the end of Chapter I. It should be emphasised from the outset that the velocity of any body will have different values depending on the definition of simultaneity, on which the time coordinate *t* clearly depends.

In addition to the change in the conception of space and time, SR's other major innovation is the assertion that the speed of light *c*, in addition to being constant in vacuum, constitutes the limiting velocity, in two senses: upper limit, for a body moving with subluminal velocity, lower limit, for a body that, hypothetically, moves with superluminal velocity. According to the Newtonian law of addition of velocities, $\mathbf{v} = \boldsymbol{u_1} + \boldsymbol{u_2}$ (where **v** is merely the sum of the two velocities**),** there is no upper limit to a body's speed. With SR, the scenario changes. This point is theoretically related to the invariance of *c* and the relativistic addition equation. Einstein derives the velocity limit, e.g. in *'Relativity: popular exposition'* from TL: « For velocity *v*=*c* one would have $\sqrt{1 - v^2/c^2} = 0$, and for even greater speeds the square root becomes imaginary. From this we conclude that in the theory of relativity the velocity *c* has the character of a limiting velocity… Naturally, this character of the velocity *c* as a limit velocity also derives from the equations of the Lorentz transformation, since these become meaningless if we choose values of *v* greater than *c*».[45] But the TLs already presuppose the constancy of *c* in vacuum. Having verified the circular character of the addition formula, one can advance the thesis that the non-composability of *c* and also, consequently, its property of maximum possible velocity, are not logically deducible and that the basis of both is exclusively experimental in character. A result of incontrovertible validity, for example, is given by the electron acceleration experiments, which demonstrate the existence of a velocity limit, however great the energy supplied. A second example of the experimental confirmation of the velocity limit is that of neutrinos (2020), which places a limit of one part on 10^{18}, to the possibility that they can exceed *c*. Roberts and Schleif, in the aforementioned study, summarise in a useful review the numerous measurements, including astrophysical ones, that attest to the constancy of *c* (Rowley, Woods, Baird, Bates, Ellis- Farakos et al, Boggs...),[46] and, in addition, the numerous experiments that confirm the speed limit *c* (Alspector and Kalbfleisch at Fermilab, Guiragosian, Greene, Stodolsky...).[47] R. Resnick notes[48] there are kinematic processes, e.g. the succession of dots on a fluorescent screen when a beam of electrons sweeps across the screen, which can have velocities greater than light, but signals or matter or energy cannot have velocities > c. These are, in short, illusory effects. There are a number of other deductive procedures in SR, both for the invariance of *c* in vacuum and for *c* limit velocity; we will analyse those that seem most motivated. There is no shortage of physicists and, of course, philosophers of science, who think one can deduce almost the entire theory from the principle of relativity alone. Among these is certainly not Einstein. There is also no small consensus on the absolutely unproven conjecture that *c* constitutes not only the universal limit velocity, but the velocity with which any object moves through spacetime; not only those of zero mass, but also massive objects, combining, in the latter case, the spatial and temporal components. For a concise discussion of the subject, I refer to the article *On Spacetime*.[49] It goes without saying that the Lorentz factor γ itself only makes sense with velocity $< c$, which places, by mathematical obviousness, an upper limit on the velocity attainable by a body, but no one could mistake this for a demonstration. There is, on the other hand, a subtle argument about the invariance of *c* , in addition to the other less well-supported ones, which requires careful examination. Light, like sound, is independent of the speed of the source. But it has no medium in which to propagate and against which its speed can be measured. Therefore, no observer has a privileged position and each observer must, regardless of his or her motion, measure the same speed as light *c*. In other words, the impossibility for an observer to measure his speed with respect to a medium would necessarily imply that his measurement of the speed of light would agree with the standard one. The point is the 'necessarily'. It is argued that observers must agree on measurements of light, otherwise they could derive the speed of their motion by reference to light. But this is impossible (for all Feynman: «There is no way to determine an absolute velocity»),[50] because it would contradict the principle of relativity. Let us examine the argument further. One can easily, without great difficulty, perform an *operational* analysis of how velocities can be measured in the reference systems S (unaccented) and S' (accented), in relative motion between them, and with or without a third, external, reference system: a) in the case of two moving cars, b) in the case of sound waves, c) in the case of light rays. It will be easily found that in the case of light rays, there are certainly valid observational reasons to affirm the constancy of *c* in vacuum, but no logical necessity. Attempts to derive the invariance of *c* as pure rational evidence turn out, on rigorous analysis, not to be consistent. It is argued: if the speed of light does not depend on the source, then there can be no other object by which it can be influenced, unless the first postulate is violated. Indeed, by measuring a different speed of light, it would be possible to measure one's own speed relative to it. Except, we must object, that this does not only apply to electromagnetic waves. The difference with sound waves is that, in such a case, one can take as a reference the medium in which the sound wave propagates. But we do not see where the a priori logical evidence is that any observer must necessarily measure for the speed of light a value exactly equal to c. *There is nothing logically* incoherent, but *nothing logically necessary* either. It can reasonably be said that none of Einstein's arguments could prove that nothing faster than light exists in the universe.

One may wonder, again, whether the principle of relativity, in the precise sense defined by Einstein, does not imply the invariance of the speed of light. Poincaré claimed that the invariance of *c* constitutes a postulate. The principle of relativity means that physical processes are independent of the IRSs in which they take place, and this can be stated more stringently: there is no way or experiment to say whether an IRS is actually at rest or in absolute motion. There is no way to determine an absolute velocity. The principle of relativity, reduced to its essential core, is the impossibility of ascertaining absolute motion. What would happen, logically speaking, if different values of the speed of light were obtained? Would the plurality of measured speeds make it possible to determine the true state of motion or stillness for different IRSs? In that case, the universality of *c* would be a logical consequence of the principle of relativity. But this is not the case. Even if an IRS were to measure, which so far does not seem to be the case, values of the speed of light different from *c* or not invariant, it could not derive from this that it is in a state of absolute motion or stillness. The correct reading is precisely the reverse, i.e. it is the invariance of *c* that legitimises the strong version of the principle of relativity. The basis, then, is always experimental, as Einstein himself explains in the 1916 text; on the principle of relativity itself we read: «... only *experience* can decide as to its correctness or incorrectness».[51] In the late 19th and early 20th century it began to become clear, with the first high-precision measurements of the speed of light, that it resulted invariant in numerous experiments of various kinds.

We are mainly investigating the logical-mathematical aspects of SR, but what emerges is the very solid experiential datum, mentioned above with some experimental indications: the speed of light in vacuum turns out to be the same in all IRSs, regardless of the relative motion of the source and the observer. Bridgman[52] emphasises that the postulate of the independence of the speed of light from its source is more indispensable in SR and much more important than that of equality in all RF. This datum has a consequence, which is self-evident: observers in motion measure different results for time intervals and lengths for any object, with the sole exception of light. Each RF has its own measure of time and space depending on where it is and how it is moving. Spatial position and time become local concepts.

A difference, perhaps the primary one, between pre-relativistic physics and the innovation introduced by Einstein, consists in the definition of simultaneity. For the former, since space and time are disjointed and time is absolute, the definition of simultaneity is immediate and in no way implies space (position, distance, speed, etc.), because it postulates the existence of an instantaneous signal that propagates with speed ∞, in the second, on the other hand, since there is a formal interdependence between spatial and temporal coordinates, it is necessary, for any synchronisation to be possible, to have an invariant velocity (not invariant time or space), and, furthermore, to define a procedure of simultaneity. Einstein uses the speed of light to set clocks. We will return to this fundamental point later. The connection with overcoming the concept of infinite speed is obvious, in the sense that the relativity of simultaneity depends on the existence of a finite speed of transmission of signals. Put another way, SR assumes that there is a standard clock at rest at each point in space and that the clocks, not being synchronised, each indicate 'local time'; the synchronisation between clocks at different points in space constitutes the definition of simultaneity. It should also be noted that for SR, which is a reciprocal theory, the results of measurements have a reciprocal nature, i.e. they can always be reversed, and that it is precisely «…disagreement about the simultaneity of events that leads to different measured values» (Resnick)[53] by IRS in relative motion. The procedures of synchronisation are by no means something ∞ to be taken for granted, as we shall see. «… there is no any instantaneous signal in nature and, therefore, the absolute simultaneity cannot be realised in any laboratory» (Y. Z. Zhang).[54] For example, are we able to synchronise, transport and directly compare the frequency values of two distant clocks? We will see, moreover, that clocks at rest at different points in space, in addition to indicating 'local time', having (and here, as we shall see below, GR comes into play) a non-identical gravitational position, beat time, with a rhythm that is not exactly the same, perhaps even to an infinitesimal extent.

In order to measure the same speed of light in all references, there is a price to pay. There is no physical basis for distinguishing between 'true' and 'apparent' motion, between 'true' and 'apparent' length, between 'true' time and 'apparent' time. Lengths and intervals of time in one IRS, measured by another IRS, given in relative motion between them, are no more 'true' than lengths and times measured by other observers in relative motion. You can't get everything for free at Einstein's restaurant! The invariant expression, however, does exist and is the spatiotemporal interval (or separation), which separates an event from the origin and which mixes space (lengths) and time (durations). We will analyse this in Chapter III.

The conclusion to be drawn is that SR, proven by an impressive amount of experimental confirmation, does not appear to be derived, at least for the side analysed here, from a logical-mathematical formalism. There are wonderful geometrical descriptions (e.g. c corresponds to the hyperbolic tangent of an angle of infinite value, so the angle remains the same, whatever quantity is added), which do not change the terms of the problem. Logic and mathematics are not sufficient to derive the speed of light. We can also call it an 'empirical law', as long as the definition of this concept is clear. This is certainly not a *diminutio*. Today's scientific culture and epistemology are mostly far from claiming that general laws can be obtained a priori from first principles. An important implication must be underlined: if the foundation of SR is experimental and not logical-mathematical, it consequently possesses a domain of validity, consisting of the empirical spatiotemporal context. «The disturbing characteristic of any empirical law is that its limits are unknown» (E. Wigner).[55] Experimental, or empirical, means subject to revision. Nor is experience a guarantee for the future. Mathematics, on the contrary, remains. This distinction constitutes a capital point for the constancy in time and space of c (see, e.g. the studies on the absorption spectra of quasars, with the hypothesis of variations in the fine structure constant α, which would mean, precisely, that the fundamental laws of physics could vary with time). We have a defined domain of validity limited to the given experiential field. It must be emphasised that when Einstein speaks of the limits of validity of SR, he is referring to a different aspect. «The theory of special relativity cannot claim to possess an unlimited domain of validity; its results are only valid as long as we can neglect the influences of gravitational fields on phenomena (e.g. on light)».[56] This is the theory's relation to inertial systems, hence it is called a 'restricted' theory, unlike General Relativity (GR), whose equations concern every body of reference, whatever its state of motion. Einstein in the years after 1905 focused on the need to «…postulate an invariance of laws even with respect to non-linear transformations of coordinates, in the four-dimensional continuum».[57]

SR therefore has a more general domain of applicability with respect to inertial systems than the Galilean-Newtonian arrangement, and it allows the form of the laws of mechanics and electromagnetism to remain invariant, but it cannot be said to have such a general domain of applicability as to embrace the universe in its entirety and in the history of its becoming, even limited to uniform rectilinear motion. This point is decidedly relevant to studies in the cosmological and astrophysical fields, as it fully encompasses our cosmic measuring rod and our cosmic clock, and consequently the scale of distances and times, with which we represent the history and state of the universe. We refer, therefore, to extrapolations on cosmological durations to present times and spatial distances beyond 'proximity'. In short, a fundamental cornerstone of physics, the universality of the speed of light, turns out to be based on experimental observation, not on logical-mathematical evidence.

Proceeding with due caution, let us check whether the foundation of SR has any logical-mathematical evidence beyond the aspects analysed so far. It must be remembered, in this regard, that the derivation of the Lorentz transformations is carried out by Einstein, as is made very clear in the 1905 text, assuming the two postulates, i.e., together with the principle of relativity, the invariance of the speed of light in vacuum, as fixed points. SR is a theory of «… the symmetry of space and time, necessary to be consistent with the principle of relativity and the invariance of *c*».[58] It is not necessary to state here the derivation of the Lorentz equations. It is clear that the transformation equations are derived from the two postulates and that the contraction of lengths, the dilation of durations and the relativity of synchronisation are consequences, in turn, of the transformation equations. The derivation of the transformation formulas is conducted with the condition that all observers (IRS) measure the same speed c for light. Space and time are bound together precisely by the Lorentz transformations.

We can arrive at similar results if, in the Lorentz transformations, we replace *c* with a lower velocity that we will call *l* . The formula will be

$$X' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{l^2}}} \qquad (10)$$

A simple examination of the Lorentz factor *Y is* sufficient. It is easy to see that if $v \geq l$ we are faced with either a division by zero or complex numbers. The condition for the formula to make mathematical or physical sense is that v is less than *l* . The transformation equation is constructed so that *c* functions as a limit quantity. But the formalism also works in the same way with any number replacing *c* .

More generally, all the consequences of the Lorentz equations (time dilation, length contraction and relativity of synchronisation, including 'phase difference' $\frac{vx'}{c^2}$) are formally derived from the two postulates, i.e. relativity and invariance of *c* , but can also be derived from the measurement processes chosen to be consistent with them. Recall that a general Lorentz transformation is equivalent to a combination of spatial rotations plus a simple Lorentz transformation along the *x-axis.*

The analysis conducted so far makes clear the relationship between the logical-mathematical apparatus and experimental data in special relativity. SR rests on an established experimental basis: the speed of light in vacuum is constant and is independent of the motion of the source and observer and the direction of propagation, in any inertial reference system. When we speak of an 'observer', we must not, of course, mean a thinking brain, but it is equally possible, as Russel said «…that it is a photographic plate or a clock».[59] There are, at present, no established and convincing experimental objections to SR, with the exception of quantum entanglement, which certainly cannot be disregarded. Accelerators in particular, with their high-energy particle collisions, and numerous cosmological phenomena, exemplarily the case of muons (Rossi-Hall experiment,[60] later confirmed with extreme precision), constitute a real test of SR, which is proven. Particle physicists experience phenomena such as time dilation every day, but it is of primary importance to clarify, as we are doing, what the basis, whether the rational evidence method or the experimental method, of the theory and its constituent parts are. There remain the problems posed by simultaneity in quantum entanglement and, also, the discussion on the collapse of the wave function (but in the latter case we are dealing not with experimental results but with a theoretical hypothesis). The former is a phenomenon of simultaneity at a distance, perhaps the most astounding of the entire Quantum Physics. Einstein constructed the thought experiment in 1935 in his tenacious effort to expose the non-completeness of QM. Simultaneous nonlocal entangled events are incompatible with SR. Entanglement thus arose as a refutation of QM's completeness. It was Schrödinger,[61] in a review of the EPR paradox, who called it entanglement. We will return to this later.

SIMULTANEITY and SYNCHRONISATION of clocks

The questions of simultaneity and synchronisation of clocks do not arise in the universe of Galileo and Newton. Galilean-Newtonian absolute simultaneity postulates the existence of a signal that propagates instantaneously at infinite speed (dates c_r the one-way velocity in the direction $\frac{\boldsymbol{r}}{r}$, c_{-r} the one-way velocity in the direction $-\frac{\boldsymbol{r}}{r}$ and C_r round-trip velocity, we have

$$c_r = c_{-r} = C_r = \infty \qquad (11)$$

In relativity, on the contrary, there is neither absolute motion or stillness, nor absolute simultaneity, nor absolute equality of time intervals, nor a signal propagating with speed ∞, nor two successive different instants referring absolutely to the same place. Any observer, equipped with clocks and measuring rods, has to measure, to get by, with these instruments the intervals of time and space respectively. Nothing could be easier? Not only at the level of common sense, but also quite frequently at the level of researches, who expertly handle the assumptions and results of SR, one comes across an underestimation of the complexity of the operation of measuring duration and length. Synchronising and measuring rods are by no means as simple operations as it would appear at first sight. Obviously, the complexity does not concern the possibility of approximate measurements that are entirely adequate FAPP (*For All Practical Purposes*), according to J. Bell's[62] expression and acronym. And if not quite for *all*, it will certainly be for *most* purposes. The question is, in principle, the exactness and, even more so, the character of the measurement . Einstein addresses this issue already in §1 of the 1905 essay (Zur Elektrodynamik....),[63] returning to it constantly in later writings, with the approach called Einstein or Poincaré-Einstein synchronisation and introducing an outline of a theory of measuring rods and clocks. This involves synchronising identical clocks in different places by means of signals and measuring the length of measuring rods in motion or even at rest. He argues that the assumptions of the existence, in principle, of (ideal, i.e. perfect) measuring rods and clocks are not independent of each other, since a light signal, reflected here and there between the ends of a rigid rod, constitutes an ideal clock, provided that the postulate of the constancy of the speed of light in vacuum does not lead to contradictions. The first part of the reasoning concerns the simplest cases: clocks at rest in an RF at rest and, similarly, the length of a rod at rest, with rod, length sample and observer all in the same RF at rest. There is no need to go over the detail. So: «... let a beam of light depart at the 'time of A' t_A from A towards B, is at the 'time of B' t_B reflected towards A and returns to A at the 'time of A' t'_A ».[64] The problem is solved, but Einstein observes that «…the two clocks *by definition* walk synchronously when $t_B - t_A = t'_A - t_B$». In the *Popular Exposition* he clarifies precisely that this would be correct if we knew, (here he uses the equivalent procedure of the midpoint M between A and B), that the speed of light on AM is equal to that on BM, but «…A verification of this hypothesis would be possible if we already had the means of measuring time at our disposal. We would therefore be said to be moving in a vicious circle».[65] In short, Einstein, even for the simplest conceptual experiment of RF at rest, does not accept, either in reference to the light clock or to the measurement of a rod, that the physical properties of light are easily verifiable. His answer to the vicious circle objection is that one must necessarily make use of *definitions* and *assumptions*, including that of the equality between unidirectional and bidirectional speed of light. I do not think it can be said any better than Einstein's own words: «...it is in reality neither an assumption nor a hypothesis about the physical nature of light, but rather a convention that I can make at my own discretion in order to arrive at a definition of simultaneity».[66] Einstein's simultaneity can be denoted as

$$c_r = c_{-r} = C_r = c \qquad (12)$$

For Einstein it is clear that the isotropy of the one-way speed of light can only be a *postulate* or *convention*. Among the first to focus on this topic we only mention M. von Laue[67] and H. Weyl,[68] authors of the so-called 'Laue-Weyl's round-trip condition' for which the time needed by a light beam to traverse a closed path of length L is $\frac{L}{c}$, where L is the length of the path and c is a constant independent of the path. The importance of this *condition* depends on the fact that time can be measured, even experimentally, with a single clock, obviously without synchronisation schemes. The one-way velocity (from a source to a detector), on the other hand, cannot be measured independently of a synchronisation *convention* of the clocks placed at the source and detector. Of quite similar sign are the analyses of Poincaré,[69] wich makes it clear that the definition of simultaneity of events in different places is a *convention* and suggests that a kind of 'unconscious opportunism' operates) and later, to mention but a few, of the prestigious philosopher of science H. Reichenbach ,[70] who considers attempts to deny the conventional nature of synchronisation almost generally refuted, and later A. Grünbaum[71] on the philosophical problems emerging from relativity. Reichenbach and Grünbaum both observe, among other interesting notations, that no observable difference would result if the speed of light were anisotropic. For M. Ruderfer[72] SR contains an important assumption, which is not tested and cannot be tested. Less relevant, from the point of view of the present work, is the well-known contribution of P. W. Bridgman,[73] who, beyond the recognised value of operational analysis and the very acute reading of SR, moves decisively within the sphere of a general philosophy, which leads him, for example, to a devaluation of GR, completely detached from the plane of experimental verification. I stress, however, that for Bridgman the specification of the concept of clock itself presents many difficulties and that its definition implies a vicious circle. The position of Reichenbach, who, in the text published in the Scientific Autobiography, edited by P. A. Schlipp[74] with Einstein's answers, states: «The fact that the simultaneity of events occurring in distant places was a conventional matter was not known, before Einstein based his particular theory of relativity on this logical discovery....The definitions of the theory of relativity are all ... coordinative definitions», where *coordinative is* equivalent to *conventional*. Reichenbach places his reading of relativity within the framework of a broad analysis of scientific knowledge. Einstein's respectful response concerns the relationship 'between the theory of relativity and philosophy' and is summarised as follows: «I can conceive of nothing more stimulating as a basis for discussion in an epistemological seminar than this short essay by Reichenbach (best taken together with Robertson's essay… In any case, it would have been *de facto* - though not theoretical – impossible for Einstein to construct the theory of general relativity if he had not accepted the objective meaning of length)».[75] Thus, in conclusion, he cut to the chase. The profound knowledge that Einstein possessed of the philosophy of knowledge and particularly of epistemological thinking also emerges in this paper, along with his awareness of the intertwining of physics and philosophy, but also his resolute adherence to the profession of physicist.

Experiments do not speak a different language, as summarised by Y. Z. Zhang's[76] already mentioned statement: «…the absolute simultaneity cannot be realised in any laboratory». The analysis of the Test Theories also shows that we are faced with different definitions of simultaneity; in particular, Einstein's and Robertson's definitions differ. As Zhang demonstrated in his 1994 paper *Test Theories of SR*, the former's simultaneity is the simplest of the theories in which «…the one-way speed' is isotropic, while the latter's is the simplest of the theories in which 'the two-way speed' is anisotropic...The definition of simultaneity can be chosen arbitrarily».[77] He draws this most general conclusion.

Taking the Poincaré-Einstein synchronisation as a basis and reference, five aspects should be emphasised:

1) simultaneity in the measurement of time requires *conventions* (that the speed of light be the same in all RF, that time be homogeneous and space homogeneous and isotropic);

2) measuring the length of a rod, even in its RF at rest, requires a synchronisation scheme, as it is essential that the two ends are measured simultaneously;

3) neither the isotropy of light, nor the homogeneity of time, nor the homogeneity and isotropy of space can be demonstrated by the described procedure;

4) Einstein states: «There is no simultaneity for events that are far apart».[78] How far apart? What is the quantitative meaning of 'immediate proximity'? The concept of local simultaneity, or 'immediate proximity', does not stand up to rigorous analysis. Whether there can be instantaneity outside of a point system would be a perfectly valid question.

5) the foundation of the concept of simultaneity is not logical-deductive, it is, on the contrary, inductive and empirical, but necessarily requires some basic *definitions/conventions/assumptions*; the insistence on this point has a strong reason: *definitions, conventions* and *assumptions* are not properties of nature, but agreements between subjects.

Point 4) becomes clearer if we add an element to the historical analyses that is fairly systematically ignored. This is the theoretical effects of the combination of SR and GR. In GR, spacetime has a non-uniform curvature, indeed extremely variable on an infinitesimal scale. The gravitational field cannot be considered perfectly identical at two different points in spacetime, as close as one wants, however imperceptible the differences may be. Thus, time flows at a different rate in regions, and even points, with different gravitational potentials. Each infinitesimal region or, even, point of spacetime has its local time corresponding to gravitational position. Thus, GR arises from SR, but then retroacts on SR and determines restrictions on it. E.g., do the concepts of 'same time' and 'same place' remain from SR? In particular, GR induces on infinitesimals, i.e. it induces the continuous. We will return to this aspect in Chapter III.

The above observations are generally valid for one-way speed of light, but not for two-way speed of light (i.e. round-trip), which requires a single clock and, therefore, no synchronisation scheme. It should, however, be noted that in the closed path with the single clock, it cannot be demonstrated that at the start-end point, the single clock always beats with the same frequency. Here again, an *assumption* is required. In any case, the two-way speed obviously cannot be used to prove the isotropy of light, the homogeneity of time and the homogeneity and isotropy of space.

The second part of Einstein's reasoning concerns several IRSs in relative motion between them. There is no need to go over his detailed analysis, because what we have seen in relation to the single RF inevitably recurs in the measurement of intervals of time or space, relative to 2 or more IRSs considered alternately at rest, and in an evidently amplified form. Observers in solidarity with a second IRS, in motion with respect to the first, will not agree on the simultaneity of events nor on the length of the rods, and clocks, if they are in motion, will not maintain synchronisation.

In conclusion, synchronisation works for clocks at rest by virtue of a definition-convention, the simultaneous measurement of the coordinates of the ends of a rigid rod is in principle not possible and, furthermore, if we apply GR, even two clocks at rest, no matter how close together, will each mark a local time, not a true time. These effects are completely negligible in an ordinary gravitational framework and at the speeds we deal with on a daily basis and become increasingly important in relation to gravitational variations and to the extent that we approach the speed of light. It can, however, be said that, strictly speaking and in principle, only two clocks in a point system would mark exactly the same time. But would two clocks in a point system be anything other than the same clock?

In Chapter II we will investigate how E. himself has set the question of the foundations of SR. We will thus be able to answer the question whether, according to the author's own interpretation of the theory, SR is based on and proceeds, and to what extent, by way of rational evidence or by way of experiment.

In Chapter III we will proceed to examine, as we did above for the equation of velocity addition, the open questions and some critical points in SR, including the concept of spacetime interval (or separation) itself, to which a section is devoted.

Finally, in Chapter IV, the conclusions of the article will be set out.

Chapter II : THE FOUNDATIONS OF SR ACCORDING TO EINSTEIN

Einstein's writings on the relationship between experience and theoretical principles in knowledge in general, and particularly in physics, are numerous and the literature on the subject is boundless. If we consider his entire output on the subject, we certainly find, depending on the work and period, abundant reasons to support different and even contradictory interpretations of his thinking on the subject. We range from the initial empiricism inspired by Hume and Mach, to later phases in which he relied more on mathematical formalism than on experience. In *'Induction and Deduction in Physics'*, as early as 1919,[79] he wrote: « ...The really great advances in our understanding of nature have been determined in a manner almost diametrically opposed to induction». It is Einstein himself who accepts (in his *'Reply to Various Authors'*, in *Scientific Autobiography*, published by P. A. Schlipp)[80] the significance of 'oscillation', responding to Henry Margenau, who had noted '…elements of rationalism and extreme empiricism': «An oscillation between these two extremes seems to me inevitable». Particularly well-argued is Reichenbach's reading, which emphasises the presence of a constant 'positivistic or, rather, empiricist orientation' (which, however, in my opinion, should not be extended to the entire span of Einstein's thought) and summarises: « Einstein's relativity thus belongs to the philosophy of empiricism».[81] In the present work, the task is fortunately for us confined to the question of the empirical or logical foundation of SR and specifically of the invariance of *c* , with particular attention to the aspect concerning the interval, or separation, of spacetime, which has the same numerical value for any observer. In Galilean-Newtonian relativity, the distance between objects in space and events in time is an absolute value, independent of the IRS, while in SR it is the new distance, constituted precisely by the spatiotemporal separation (or interval), that is invariant and has an absolute value.

All objects go at different speeds, which can be added together and vary with respect to us, depending on the state of motion or which reference system is used to observe it. This says the Galilean-Newtonian law of addition of velocities. Einstein's revolution is that light is an exception. It is neither additive, nor variable, nor dependent on source and sr. He takes the conflict between classical mechanics and Maxwell head-on and overcomes the mechanistic criterion and the traditional view of time and space, introducing new kinematics and dynamics. Maxwell, in part, and Einstein, in full, thus revolutionised our conception of time and space. In changing the reference system, it is not the speed of light that varies, but spatial and temporal measurements.

Einstein's texts show that the invariance of *c*, this 'absurd axiom', according to Pauli's expression, is sometimes introduced primarily as a postulate, sometimes as the result of experimental measurements, sometimes together as a postulate and as experience. The process of his thinking is evident from two fundamental texts: the disruptive essay of the annus mirabilis 1905, *'Zur Elektrodynamik bewegter Korper'*, and *'Uber die spezielle und allgemeine Relativitatstheorie'* of 1916.[82] The term 'postulate' is in present-day logic mainly used as a synonym for axiom (self-evident proposition), but was and is also often used as an assertion assumed to be true in order to arrive at the truth of a thesis. Postulates in physics, however, are predominantly defined as not depending only on the consistency of the formal system.

Einstein declares that the law of propagation of light in a vacuum, combined with the principle of relativity, forms the basis of the new theory. This provides a new foundation for mechanics. It actually contains three different aspects: a) light propagates with finite speed; b) it is independent of the motion of the source and RF's; c) it constitutes the limiting speed.

«The relations must be chosen in such a way that the law of propagation of light in vacuum is satisfied for the same ray of light (and of course for every ray) with respect to both *K* and K'».[83] And again, arguing the equality of the transmission speed of light with respect to different bodies of reference: «Of course we should not be surprised at this, since the equations of the Lorentz transformation were derived precisely in accordance with this point of view».[84] Feynman comments clearly: «This is good, for it is, in fact, what the Einstein theory of relativity was designed to do in the first place - so it had better work! ».[85] The transformation equations are derived by holding constant the propagation velocity *c* with respect to any reference system. That is, they assume *c* to be constant and make the coordinates of space and time vary. If the speed of light is constant in all RF, clocks must change the rate and rulers the length. To satisfy the postulates of relativity, the transformation equations must guarantee that a light signal propagating in the positive x direction and having in S universe line of equation $x = ct$, therefore $x - ct = 0$, has in S' universe line $x'-ct'= 0$. The postulate of relativity also requires that light propagating in the negative *x direction* in *S*, i.e.

$$\text{a) } -x = ct \quad \text{or} \quad \text{b) } x + ct = 0 \tag{13}$$

has velocity c also observed in S'. These equations are valid for the transition from one inertial system to another in uniform motion relative to the former. The derivation of the transformations is carried out, in conclusion, assuming three assumptions: the principle of relativity, the homogeneity of space and time, i.e. all points in space and time are equivalent (this is flat space, without mass and without matter) and the invariance of the speed of light in vacuum in all inertial systems. As Planck summarises exemplarily: «The speed of light is the central absolute datum of the theory of relativity».[86]

Another passage, also from the 1916 essay, reads: «Experience has led to the conviction that, on the one hand, the principle of relativity (in the restricted sense) is valid, and that, on the other hand, the speed of propagation of light in vacuum must be considered equal to a constant *c*. By combining these two postulates, we have obtained the law of transformation... the Lorentz transformation. In this course of thought, the law of propagation of light played an important part, the acceptance of which is justified by our actual knowledge… If a general law of nature were to be found that did not satisfy this condition, then at least one of the two fundamental hypotheses of the theory would be contradicted».[87]

From these texts, Einstein's approach to the basis of the principle of relativity and the law of light propagation, defined in the 1916 work as «…these two fundamental results of experience», is clear. The two postulates, as W. Pauli notes, in his Relativitätstheorie 1921,[88] are unified in the condition of invariance of all laws of nature with respect to the Lorentz group. To be more precise, rather than constancy of the speed of light, it would be better to speak, Pauli again notes in the same cited work, of *'independence of the state of motion of the source'*, because this also exists in general relativity, i.e. not only in inertial reference systems.

Thus, the new theory is based on two postulates: 1) the laws of physics have the same form in any inertial reference; 2) the speed of light is the same in any inertial reference, whether it is emitted by a body at rest or by a body in uniform motion. It should be emphasised that the other assumptions are homogeneity and isotropy (invariance in space for translations or rotations) and homogeneity (invariance for translations in time) of spacetime.

It is clear that for Einstein these are two postulates that are empirically based, arise from observations, are not contradicted by any observations, and allow for a coherent and complete theoretical accommodation of the data of experience in kinematics and electrodynamics. Logical deductions confirmed by empirical evidence. It can be said that for Einstein, what is most important is coherence combined with the ability to describe the phenomena of the entire theoretical and experimental setup. He considered, for example, the unification of the law of conservation of energy and the relativistic reformulation of conservation of mass to be the most important general result of SR.

It has been mentioned that there is a vast literature on the relationship in Einstein between empirical observation and mathematical reflection. Abraham Pais notes in his *'Subtle is the Lord...«...* that he was inclined to rely on purely logical and mathematical arguments and that the June 1905 memoir itself has the axiomatic structure typical of a complete theory».[89] Einstein, in fact, frequently adopts a deductive procedure.

In the 1933 Oxford lecture *On the Method of Theoretical Physics* he said: «I am convinced that by means of purely mathematical constructions it is possible to discover those concepts that give us the key to understanding natural phenomena and the principles that bind them together».[90] So he expresses different positions on the relationship between theory and observation at different times, but on the invariance of *c* historical genesis and epistemological reflection are unambiguous. It must be emphasised, however, that for the maturation of SR the decisive point was not the Michelson-Morley[91] experiment, which historically had a diriment value for the overcoming of the ether and instead seems to have only indirectly influenced Einsteinian elaboration, but rather the contrast between classical mechanics and Maxwell's electrodynamics. It was this that made a new theory necessary. It is plausible that Einstein arrived at SR almost only by examining the logical flaws in the then current physical theories. Nevertheless, it is clear that he was well aware that experimental control can sweep away any theory, no matter how mathematically elegant, simple and consistent. A single experiment can demolish a theory. He had read and pondered in depth thinkers such as David Hume and Ernst Mach, who believed that one must rely on observations alone. Bertrand Russel in The ABC of Relativity, of 1925, summarises: «Nevertheless, it must not be forgotten that the experimental results were at the origin of the whole theory and were the starting point from which began that tremendous undertaking of logical reconstruction which is implicit in Einstein's theories».[92]

It appears from the texts that the two postulates are assumed by Einstein not because of any logical-mathematical evidence, but because they are confirmed without exception by experimental data. He is even clearer in paragraph 13 of the 1916 work, writing, in explicit reference to the two formulae of the addition of velocities, traditional and relativistic : « The problem now arises of deciding which of these two theorems agrees better with experience....... The experiment is resolved in favour of the equation derived from the theory of relativity».[93] In short, it is experience that validates or refutes.

In paragraph 18 of chapter 2 of the same 1916 text, he exhaustively clarifies the terms of the question. It is a necessarily valid a priori assertion that motion is exclusively relative and that one can choose for the description of any event the body of reference one prefers. As regards, on the other hand, the principle of relativity in its 'broader sense', i.e. the assertion that no one reference system is privileged in comparison with another and that the laws of mechanics and the law of the propagation of light have exactly the same formulation, «…only experience can decide on its correctness or incorrectness».[94]

On the relationship between theory and observation in Einstein, it is illuminating to reread the popularisation work of 1938, written with Leopold Infeld.[95] The popular character is not at the expense of rigour, and an extraordinarily effective picture of the development of physics, from Newton to Maxwell, relativity and the quanta, emerges. In the same vein, of great interest is the Scientific Autobiography, compiled by P. A. Schlipp,[96] with various contributions, (Pauli ,[97] Born ,[98] Heitler ,[99] Bohr ,[100] Margenau ,[101] Reichenbach,[102] Gödel)[103] and Einstein's reply to the observations.[104]

Beyond the question of the basis of the theory, the experimental data on SR are, as is widely acknowledged, beyond dispute. The experimental framework is more than well known. SR comes after a long theoretical and experimental gestation. It is sufficient to recall on the level of theoretical elaboration Maxwell, Hertz, Larmor, Voight, Lorentz, Poincaré, FitzGerald .... and on the experimental level Fizeau, Michelson-Morley, the phenomenon of aberration, astronomical observation of double stars, the increase in mass with velocity and the empirical observations made in the following decades, and still today, by astrophysical phenomena and accelerator science. Attempts to maintain the concept of the privileged ether reference soon proved that there was no experimental basis to support the privileged RF.

In this Chapter we have tried to understand Einstein's viewpoint on the subject; however one interprets it, what emerges is that, in the final analysis, it is experience that decides. The invariance of *c*, or rather the independence of source motion and RF, and the principle of relativity, in the extended sense of the new theory, have a higher degree of empirical corroboration than most of the theories we experience. Starting from experimental confirmation is the basis for anyone who wants to reason solidly about SR. Therein lies its very foundation. Similarly, it should be assumed that the invariance of *c* in vacuum belongs not to the world of logical truth, but to the world of empirical data. The speed of light is an argument from experimental physics. And it is not at all negligible that what has an experimental character, and consequently, a defined domain of applicability, should be recognised as such. For everything that is empirical is subject to revocation, should the experimental data require it. As a conclusion to this Chapter, it is useful to quote the passage from *Scientific Autobiography*, in which, referring to the theory of light underlying SR and GR, we read «…one implicitly makes use of the existence of an *arbitrarily* defined optical signal».[105] One of the many demonstrations of our author's truly rare attitude of freedom with regard to his own theory.

Chapter III : ON SOME OPEN QUESTIONS IN SR

It must be stated at the outset that *'open questions'* are not refutations expressed in a polite tone, they are instead, precisely, open questions. The difficulties of a theory can push us deeper than its own results.

Reflection on SR still remains one of the most important areas of physics. Indeed, the theory of relativity, both restricted and general, constitutes the best theoretical-experimental framework, available to us, for the description of spatial and temporal relations. Some of the problems Einstein tackled still constitute a formidable theoretical and experimental challenge today.

Any theory of space and time needs measurements, i.e. clocks and measuring rods. Whatever they may be, with rigid bodies, with light signals or of any other kind. A clock is an instrument, natural or not, for computing equal intervals of time based on a periodic phenomenon. Conceptually, a pendulum clock and an atomic clock are equivalent. Time, by the way, is the quantity that is measured better than any other. It is now possible to measure time intervals with an uncertainty of the order of 10^{-18} (one part in a billion billion). The devices, interval resolution and precision change, but not the function. A sample measuring rod, a traditional instrument for measuring length, needs (to avoid having to use another rod and then another) a sample of length, for example a wavelength, or a clock and an object moving with uniform velocity, since $s = t\,v$. To test SR we can indifferently use the instruments of Einstein's time or those of our own, such as an atomic clock and transitions between energy states. There have been spectacular experimental advances in precision metrology and high-precision resolution of ultrafast processes in the domains of space and time, e.g. optical autocorrelation and the quantum pulse gate. New technologies and instrumentation have not, so far, refuted special relativity in the slightest; on the contrary, they have confirmed it. I think it can be shown that the questions that are still open were essentially such and detectable already in the original Gedankenexperiments.

We discussed in Chapter I the incompatibility between the principle of relativity in its pre-relativistic formulation and electromagnetic theory, the conflict between them - the classical addition of velocities cannot be reconciled with the invariance of *c* - and the two possible ways of overcoming it: either a privileged IRS (the aether) or a modification of classical mechanics.

«The strength of the new theory lies in the consistency and simplicity with which all difficulties are resolved by resorting to only a few plausible assumptions».[106] The ability to solve difficulties is undeniable, SR explains phenomena that could not be explained in Newtonian physics, which considered time a universal coordinate valid for all observers. Yet, it can be shown that not all accounts add up. I am certainly not going to lengthen the conspicuous list of unfortunates who have unsuccessfully tried to outwit old Einstein. However, questions remain on the table, about the measurement of space and time and about spatial and temporal relations in general. A number of conceptual difficulties persist in special relativity, which need to be re-examined. The thorny questions and this work may or may not hit the mark, but we will see something subtle and unexpected emerge.

1) Firstly, as has been shown (Chapter I), the equation of addition of velocities has a circular character. Its formalism has no demonstrative efficacy; instead, the real 'demonstration' is of an experimental nature.
2) The invariance of *c*, the cornerstone of the theory, is also based on observations, not logical-mathematical evidence.
3) The speed limit *c*, too, can be reasoned in a similar way. That light has a finite speed, and that *c* constitutes the speed limit, has been experimentally verified and beyond dispute for more than a century. That nature has a speed limit has profound consequences: firstly, it forces us to set aside the idea that time and the same order

of events are the same for all observers, and secondly, it implies that superluminal processes constitute a violation of causality. Einstein states that any motion with infinite speed is not rationally conceivable. Furthermore, the fact that the speed of light, apart from being finite, constitutes the limiting speed is supported by a great deal of experimental evidence. However, it has not been deduced, i.e. logically proven, and it can also be reasonably argued that it is not deducible and provable. Like the invariance of *c*, the barrier of light, limit velocity, belongs to experimental physics, which so far confirms SR. A classic example is the kinetic energy equation:

$$K = m_0c^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right] \qquad (14)$$

where for $u \rightarrow c$ K tends to ∞. The Lorentz factor is certainly not sufficient for logical evidence, so that necessarily $v < c$. However, what is experimentally proven is that one cannot reach the limiting velocity from lower velocities; as for velocities above c, one would first have to find them! I stress, however, that many physicists, theoretical and experimental, do well to be very much *on the alert* about the experimental work going on in this area. For example, experiments on the acceleration and deceleration of spatiotemporal optical wave packets, e.g. pulsed laser beams and group velocity of light pulses, are worth mentioning. As already seen in Chapter I, it is also of great interest to read the publications on recent experiments, of various approaches, in the field of LIV (Lorentz Invariance Violation),[107] including tests on the energy dependence of the speed of light in vacuum and the GRB experiments (with the latter, no LIV has been observed, as M. Doro notes, «...it may simply come further than where we have been looking so far»).[108]

4) Among the open questions in SR is the concept of an inertial system, IRS (inertial reference frame or inertial system of reference). Any inertial reference system is a lattice of rulers, chosen arbitrarily from an infinite number of options, plus a (non-finite) set of clocks assumed to be synchronised, arranged at each point in space, within the sr. The principle of inertia, which underlies Newtonian mechanics, postulates the existence of IRSs, and the starting point of SR is also that IRSs can be identified. These are defined as the RF where Newton's 1st law is valid. Non-accelerated bodies, subject to an overall zero external force, that remain in the state of stillness or uniform rectilinear motion and, in addition, all systems in rectilinear motion with uniform velocity with respect to the former. The definition of IRS implies, in addition to the conformity of Newton's 1st law, the applicability of Euclidean geometry and a notion of time valid for the entire SR, i.e. the *t-coordinate* represents a common time in the entire reference system considered. The notion of IRS implies, therefore, a definition of the time coordinate, i.e. a definition of simultaneity. We have seen (Chapter I) that there are different definitions of simultaneity, even in the field of SR. The theory affirms the existence of a class of special observers, the inertial observers, and posits the transformation rules that apply between their observations. Observer is to be understood, clearly, in a broad sense, including the biological dimension and technological apparatuses. We can take Resnick's[109] definition : «An observer is actually an infinite system of recording clocks distributed throughout space, at rest and synchronised with respect to each other». The IRS can be thought of as a conventionally at rest and synchronised system. A system, moreover, in which

there is an identity of measurements, in which all measuring rods measure the same distances, all clocks beat with the same rhythm and mark the same time. It was J. Thomson[110] who, in 1884, introduced the concepts of reference system and reference clock, i.e. a spatial and temporal frame against which motion could be measured, instead of using absolute space and time. In 1885, L. Lange[111] adopted the expressions of inertial system and inertial time scale. Thus matured, also with Mach's[112] strong contribution, the idea that mechanics does not need absolute space and time, but is sufficient, in Einstein's words, a coordinate system in which the equations of mechanics apply. In relativity, inertial systems are equivalent. If there were an inertial observer, moreover, there would logically be an infinite number of them. As the reference systems, from which events are observed, vary, so do the spatial and temporal measures and, with them, the length of time intervals and the length of distances. Every non-inertial RF has its ruler and its clock, or rather its rulers and its clocks. A clock at every point in space. And a ruler, or if you prefer a metric, at every point in space. IRSs, as bodies on which no force acts, however, would seem to be able to replace Newton's absolute time and space, in a certain sense, as the basis for mechanics. Einstein himself, however, wonders, for example in *The Evolution of Physics,*[113] whether observers and inertial systems really exist. He speaks of the IRS as something *'mysterious'*, in virtue of which absolute motion and absolute stillness would make sense. Again, he writes[114]: «The weak point of the principle of inertia is that it implies a circular argument: a mass moves without acceleration if it is sufficiently distant from other bodies; but we know that it is sufficiently distant from other bodies only by the fact that it moves without acceleration ». The conclusion he reaches is that an RF can only be thought of as inertial locally, i.e. limited and circumscribed, because the inertial character is limited in space and time. The so-called 'inertial time' is the time scale with respect to which objects (particles) move in uniform rectilinear motion. With this understanding, Einstein can state that «…the phantoms 'absolute motion' and 'absolute inertial system' can be expelled».[115]

5) Another point concerns the symmetrical and reciprocal character of IRSs. They are, for the theory, equivalent, symmetrical and indistinguishable. Poincaré[116] went so far as to advance the conjecture, in *'Science and Hypothesis'*, that a violation of the principle of relativity, even if it occurred, could never be detected. There is no absolute RF at rest that is privileged over any other. It must, however, be emphasised that, for SR, the proper length (in the reference at rest) constitutes the maximum possible length for an object and that, in a related manner, the proper time, i.e. the time measured by a clock at rest in its RF, possesses the greater frequency (proper frequency) than the time measured, in any other IRS in motion with respect to it, by clocks integral with it, which lag by the Lorentz factor. The measurements of l_0 e t_0 as well as m_0, remain invariant in all IRSs. The only *stable measures* and predictions, on which all observers agree, are those of the RF considered, from time to time, at rest, while all other measures are *unstable.* The measurements will, therefore, be *stable* or *unstable* for observers in different IRSs. We will see below (Chapter III, point 8) that we are also faced with genuine paradoxical instabilities. Symmetry and reciprocity are not denied, but the invariance of measurements at rest would require a more convincing explanation in SR, which is a reciprocal theory. The principle of relativity should

not contain anything that attributes a 'preferred position' to any RF. The question to be asked is whether there is not a preferred IRS, in some sense, over all others. Certainly, all IRSs, alternatively, when assumed to be at rest possess this property, but, nevertheless, the reference at rest can be interpreted as a kind of special reference with respect to all other sr. The three values, observed in any IRS, considered in a state of rest, are quantities on which all sympathetic observers agree, i.e. they are invariants of the theory. All IRSs are equally entitled to assume their metre and clock as being at rest. It is often erroneously repeated that there are only two invariant quantities in SR, namely c and the spacetime interval, but we have just seen that this is not the case. The quantities of time, length and mass, measured in the proper reference, i.e. in any IRS legitimately assumed to be at rest, are also invariant. In essence, we are noting that τ, l_0 e m_0 are absolute quantities, valid in the reference at rest for all observers. There is, moreover, another aspect: two clocks placed at the ends of any non-point object in motion do not allow for exact simultaneity and pose a problem of synchronisation, which can be solved in practice, FAPP, but not theoretically. Put more precisely: the determination of $l_{0,}$ t_0 e m_0 carries with it, to be rigorous, the whole problem of length and duration measurements in SR. If we take the case of a rod at rest, the measurement is defined as the distance between the two extremes measured simultaneously and simultaneity is no less possible, as we have seen, than in the case of a rod in motion. Two events, in fact, at points P and Q of an IRS, are simultaneous if, and only if, the light emanating from them arrives at the centre of segment PQ at exactly the same instant. We have shown above that this requires, not only observations and measurements, but also *assumptions/conventions*. The problem, in short, is that the relativity of simultaneity is not only valid in relation to different IRSs, but also for the single IRS, in motion and even at rest, if the rules of SR are strictly applied. For the sake of clarity, the time coordinate represents, as also Zhang notes, «…a common time within the whole system…»[117] of reference, but this, we add, is not in virtue of logical evidence or empirical fact, it is instead an *assumption* or *definition* or *convention*.

6) SR places mathematical and physical restrictions (i.e. constraints) on all processes occurring in time and on all physical theories dealing with space and time. Thus SR is a genuine scientific theory, exposed to experimental disproof and falsifiability. The primary restriction is the finite nature of the propagation speed of all known physical effects. It is a very challenging and very demanding theory, because the restrictions are not insignificant. Perhaps the most notable issue is posed by entanglement, which, after the theoretical and/or experimental work of physicists of the stature of Bohm,[118] Bell,[119] Shimony,[120] Clauser,[121]Horne,[122] Aspect,[123] Zeilinger,[124] Greenberger[125] and others, seems to be assumed to be experimentally established. Bell's inequalities and the experiments generated by them, called by many researchers, following Shimony, an '*experimental metaphysics*', pose SR with a fundamental question. Nature seems to lack the constraint of locality and, moreover, the simultaneity of the effects of quantum correlation appears up to now to be tested. One cannot, therefore, escape the question of whether and to what extent we have a satisfactory theory of spatial and temporal relations. In any case, there seems to be a *manifestly essential conflict* between quantum physics and relativity.

7) *THE SPACE-TEMPORAL INTERVAL (or SEPARATION) AND PROPER TIME.* We owe to Hermann Minkowski (1909), and to Poincaré (as early as 1906), the introduction of the uniform treatment of the coordinates of space and time and, therefore, the four-dimensional geometry of spacetime and the concept of time as a dimension of four-dimensional spacetime (a 4-variety). Space and time appear to mix and become interdependent with the application of Lorentz transformations. The geometry of spacetime, symmetry based on rotations in 4 dimensions, is very different from 3-dimensional geometry. In the view of classical physics, space and time were absolute and independent. Relativity, on the other hand, usually warps time and space and merges them. We experience, however, that time, unlike the spatial components, has constant direction. It is one-way. It can only do one thing: flow forward. It is legitimate to ask how space and time can, in SR, turn into each other and vice versa, the temporal coordinate not being perfectly homogeneous with the spatial dimensions. In fact, we operate on time as if it were an additional spatial dimension. The mechanism is analogous to what happens with the conservation of energy, where it is not the kinetic energy K and the energy at rest mc^2 separately, but rather

$$\sum K + \sum m c^2 \qquad (15)$$

In the latter case, however, it is a question of kinetic energy and energy at rest, in the other case of space and time, the homogeneity of which is neither taken for granted nor demonstrated. Time cannot move, unlike bodies, back and forth; once the origin is fixed, the interval is always $\Delta t > 0$. Even for SR, time is not, however, precisely on the same plane with respect to spatial coordinates. There is a difference in sign. The problem is, therefore, to have a mathematical formalism capable of describing the new combination of space and time, brought back to a single entity. The first solution, particularly in the initial phase, is to take the same dimensions for the coordinates by multiplying time by the speed of light, *ct,* thus obtaining a length, homogeneous with the spatial coordinates. Having done this, with the multiplication by *i* one also obtains the indispensable sign difference. It is, in fact, mathematically necessary that spatial and temporal components have opposite signs. The minus sign is indispensable, yet one may wonder whether it is not gratuitous or even arbitrary. Nor is it forbidden to ask whether the gain in dimensional homogeneity is sufficient to justify the transformation of the temporal coordinate, unidirectional and 'one-way movement', into the spatial coordinate, with infinite possible directions and 'two-way movement'. Starting with Minkowski[126] himself, and then in later years, different formalisms were elaborated, especially with the development of quadri-vector and quadri-tensor mathematics, also in connection with the refinement of topological thinking. In this framework, the solution consists essentially in endowing the vector space with a scalar product (and consequently a distance) different from the canonical Euclidean space, in which the scalar product is defined as positive, unlike in pseudo-Euclidean space. More precisely, Minkowski's spacetime is an affine space of dimension 4, endowed with a scalar product with signature (3.1; - +++) or also, equivalently, (1.3; + - - -), non-degenerate, but not defined positive. Thus, the sign is positive or negative depending on the conventions on g_{00} (0 being the time coordinate), with opposite sign to the other 3 coordinates. Let us express, for the sake of clarity, the scalar vector with sign (3,1) :

$$< (x_0, x_1, x_2, x_3), (y_0, y_1, y_2, y_3) > = -x_0 y_0 + x_1 y_1 + x_2 y_2 + x_3 y_3 \qquad (16)$$

In my article, being published, 'On spacetime', there is an in-depth discussion of this aspect. Minkowski's formalisation was fully accepted by Einstein, after initial perplexity as well as some irony (he is said to have commented : "I don't understand it any more either"). In it, coordinates are parameterised not in relation to the time coordinate, which varies with the reference system in which it is measured, but as a function of proper time, which is invariant. «A proper time interval is in contrast to the coordinate time interval, which does not depend upon the definition of simultaneity» .[127] Four-vectors velocity generalises the 3-dimensional velocity (the four-vectors generalise the ordinary tri-vectors of classical mechanics), based on the variation of spatial and temporal coordinates with respect to proper time τ. The quantity τ introduced by the theory of relativity, is an invariant in any IRS, since every observer, integral to a given IRS, measures a constant proper time. The spatiotemporal interval (or separation), i.e. the distance of an event from the origin, which constitutes the invariant expression in the passage, in accordance with the Lorentz transformations, from one IRS to another, is a quantity related also to proper time. In summary:

$$t^2 - x^2 = t'^2 - x'^2 = \tau^2 \qquad (17)$$

the square root of the last member is precisely the proper time τ. The magnitude τ, whose formula is

$$(\tau^2 = t^2 - x^2 - y^2 - z^2) \qquad (18)$$

turns out to be the opposite of the spacetime interval $\Delta(s)$, in fact

$$(s^2 = -t^2 + x^2 + y^2 + z^2) \qquad (19)$$

These are invariant quantities for Lorentz transformations, i.e. for any combination of boosts and rotations. This is the most important concept in relativity. It can also be written in the tensor form

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \qquad (20)$$

Tensor language is elegant and stimulating, but, unlike in GR, in reference to SR it is not indispensable and we are not sure that it makes things more comprehensible.

The abbreviated formulas are:

$$a) \quad \tau^2 = t^2 - x^2; \quad b) \quad s^2 = -t^2 + x^2 \qquad (21)$$

(with *x* vector in both). The fact that the spatiotemporal interval is independent of the RF in which the events are observed and that it is equal, with opposite sign, to the proper time, i.e. measured in the reference with which the body or event considered is integral, has an apparently simple explanation: by combining the invariance for rotations of the spatial coordinates x, *y*, *z* (for the three-dimensional version of the Pythagorean theorem $x^2 + y^2 + z^2$ does not vary for spatial rotations) and a rotation of the spatial coordinates, which has no effect on time, one obtains both the invariance of proper time τ and the equation

$$t^2 - x^2 = t'^2 - x'^2 \qquad (22)$$

It is logical to ask, however, what is the relationship, both mathematical and physical, between proper time τ and the relativistic interval $\Delta(s)$. Depending on whether one operates with the [-, +, +, +] or [+, -, -, -] sign, the equations give either the spacetime interval or proper time. Now, interval is an objective relation between events, one might say a property, and proper time is, in turn, an objective and invariant character of events in any IRS considered at rest. Why do these two objective data coincide, except for the change of sign? It cannot, of course, be a coincidence of measurements on different objects, nor even a simple mathematical expedient to make the accounts add up. This leads, rather, to trying to verify whether it could be, as is reasonable to assume, mathematical identities derived from the definitions: a) that of proper time [eq. n.(18) and n.(21)]; b) that of interval, which is obtained by the root of the difference between the squares of the distance travelled by light in the temporal separation and the spatial separation. Let us take a closer look. In the IRS *R*,

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2; \quad (\Delta\tau)^2 = (\Delta t)^2 - (\Delta x)^2 \qquad (23)$$

It is evident that $(\Delta x)^2$ - or even Δx^{i2} - is just another way of writing $(\Delta x^2 + \Delta y^2 + \Delta z^2)$. Let us take the case where the spatial coordinates, with respect to RF, considered at rest are zero, $(\Delta x)^2=0$. You will have

$$(\Delta s)^2 = -(\Delta t)^2 \; ; \quad (\Delta\tau)^2 = (\Delta t)^2 \qquad (24)$$

i.e., in the reference of stillness, the proper time τ and time *t* are coincident. The interval, an invariant quantity in all IRSs, is none other than the distance from the origin O, i.e. the distance travelled by light in the time considered ($x_0 = c\, t_0$), which is precisely τ. This, therefore, with $(x^2 + y^2 + z^2) = 0$, while (since in the RF proper, of course, one can measure the spatial components of a phenomenon in addition to the duration) with $(x^2 + y^2 + z^2 \neq 0)$ the interval will be the difference between time *t* and the sum of the spatial coordinates, according to the Lorentz transformations. One can, of course, derive the coordinates taken from a clock integral with an IRS *R′* in uniform rectilinear motion with respect to *R*, but the proper time is by definition the time measured in an RF integral with the measured phenomenon. To put it, effectively, with Resnick, the clock that measures τ, the proper time, "is always at the place where each event occurs". The interval, which is an invariant quantity in all IRSs, in fact is that of [eq. n.(19)], turns out to be, as above, the distance from the origin O, i.e. the distance travelled by light in time *t*. But *t* (the coordinate time interval) is not τ, at relativistic velocities we have $t \neq \tau$. Proper time is measured by a single clock integral with the primary system, whereas transformed time requires (at least) a second clock in the new system. The proper time (τ) and the relativistic interval (Δs) are, in accordance with the hypothesis to be tested, the same concept, i.e. mathematical identities derived from definitions. Proper time is exactly the interval, the difference of the squares of the temporal and spatial components, changed by sign. Recall that proper time, i.e. the time measured by a single clock in its RF at rest, measures an invariant time interval, but different from the measurements of any other moving clock. The separation between two events is equal to the proper time of the event measured in the transformed system. Relativistic interval Δs and proper time $\Delta\tau$ are invariant scalars for Lorentz transformations . The proper time-interval relation also has another very elegant geometric representation, as it is described by a curve, the hyperbola, which is a structure of admirable symmetries. In fact, given two IRS *S* , *S′*, in relative motion at any velocity: a) the Lorentz equations transform an orthogonal system into a non-orthogonal one; b) the angle between the spatial axes will be equal to the angle between the temporal axes; c) the calibration curves will give, on the axes of references, the unit length and the unit time interval. To appreciate their meaning and beauty, just draw the two branches of the hyperbola

$$\omega^2 - x^2 = 1 \qquad (25)$$

and the two branches of the hyperbola

$$x^2 - \omega^2 = 1 \qquad (26)$$

If we take the formula of the spacetime separation

$$\omega^2 - x^2 = s^2 \qquad (27)$$

and we invert w and x, we obtain the substitution of s^2 with $-s^2$, which is equal to the square of the proper time τ. What we experience, in each case, is that the two geometries are representative modes of the same formalism and that the formalism works in describing the observations. However, by conducting a careful analysis of the mathematical procedure, we show that the relation between τ^2 (the square of proper time) and s^2(the square of the interval) implies an ill-defined and, indeed, ambivalent use, as argued above, of the concept of proper time. Let us elaborate further: let us consider, in the standard way, two inertial IRS with R' and R in relative translatory uniform rectilinear motion, with constant velocity v. Let us take on R the two equations: the first is [eq.n.(19)] and the second is [eq.n.(18)]. If the sum of the spatial coordinates $x + y + z = 0$, we will have

$$a) \quad s^2 = -t^2 \ ; \quad b) \quad \tau^2 = t^2 \qquad (28)$$

and if, on the other hand, their sum $x + y + z \neq 0$, we will have different results, specifically in the second equation, since τ^2 will not be equal to the square of the time coordinate, but rather the difference between the square of the time coordinate and the square of the sum of the spatial coordinates. The same applies to measurements made assuming the system R' as the reference of rest. In other words, we are pointing out that different formal constructs always contain τ, but with sometimes different valence and meaning. There are not a few authors who sometimes do not distinguish exactly between proper time and time coordinate. Clearly, these are measurements taken in/from two different IRSs, the original and the transformed system, which moreover can always be reversed in full reciprocity. Proper time τ does not require spatial coordinates x_i to be defined, unlike the time coordinate t. When, however, τ is a term in an equation in which spatial coordinates appear, such as $\tau^2 = t^2 - x^2$ with any combination of temporal and spatial coordinates, except of course in the case $x = 0$, it means that we are using proper time τ in one case and, in the other case, the same time interval, but seen from another IRS and measured, correctly, according to the Lorentz transformations ($t = \gamma\tau$). The new equation,

$$\Delta t = \Delta t_0 / \sqrt{1 - \beta^2} \qquad (29)$$

gives the relation between the time coordinate and the proper time. Obviously, the quotient between the proper time and the time coordinate gives the Lorentz factor :

$$\frac{d\tau}{dt} = \sqrt{1 - \beta^2} \qquad (30)$$

which in the case $v = c$ is equal to zero, in the case $v > c$ gives a complex number and only in the case $v < c$, gives a result with meaning. Skipping the otherwise simple and obvious steps here, we obtain

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$$\tau = t\left(\sqrt{1-\beta^2}\right) \qquad \underline{(31)}$$

which, assuming R at rest and R' considered in motion with respect to it, having to express x and t as a function of x' and t', generates some problems, as we see immediately. To verify this, elementary algebra suffices. Let us take two equations, found in almost every textbook:

$$\text{a) } t = \gamma\tau\ ; \quad \text{b) } \tau = ct \qquad (32)$$

The second, one might observe, presents a problem of dimensional homogeneity, in fact, of the two members of the equality, one is a time and the other a length, therefore $[\,t\,] = [\,L\,]$. The dimensional analysis, by the way, captures a real datum, as the time coordinate is treated in the formalism as a distance. However, if we combine the two equations, we will have

$$t = \gamma ct \qquad (33)$$

which, apart from having the same dimensional inconsistency, makes no logical sense, and by carrying it out , with $v <, =, >$ with respect to c, would in all cases lead to some results that make no mathematical and/or physical sense at all. Again starting from equations a) and b), one can immediately obtain

$$\tau = c\gamma\tau \qquad (34)$$

which presents the same problem and also requires ingenious interpretations. Is there any mathematical fallacy in this analysis? The fact is that the procedure usually adopted appears a bit dirty. Things that do not add up should not be overlooked.

We can reach the same conclusion by a much simpler and more direct route. Let us consider, within the framework of Minkowski's metric, two events P,Q . Let us consider two cases: a) P and Q occur in the same IRS, with which the only clock involved is integral; b) the spatial and/or temporal distance (P,Q) is measured by an IRS S' in relative motion with respect to S. The time taken by a ray of light to travel the distance (P,Q) will be $\Delta t=\frac{\Delta S}{c}$. Hence $\Delta s=c\Delta t$ and by squaring and shifting to the first member we will have

$$\Delta s^2 - c^2\Delta t^2 = 0 \qquad (35)$$

Substituting Δs^2 with the spatial coordinates and inverting the sign gives

$$c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = 0 \qquad (36)$$

The square of the distance travelled by light is equal to the sum of the squares of the spatial coordinates.

In an even clearer way:

$$x^2 + y^2 + z^2 - (ct)^2 = x'^2 + y'^2 + z'^2 - (ct')^2 = 0 \qquad (37)$$

That is, the sum of the spatial coordinates is, in both equations, equal to the temporal coordinates.

Also on the question of the change of sign, whether this is achieved through multiplication by *i* or through a suitable choice of metric space and type of scalar product (in the metric tensor, e.g., the sign can be defined as positive with *n,0,0* or negative with *n-1,1,0*), it can be concluded that these procedures undoubtedly adopt correct mathematical options in the search for patterns of description of spatial and temporal phenomena, but also that they do not seem to be able to constitute proof of any theory, among those considered, of space and time.

One can now well understand, by virtue of the analysis conducted, *why the interval coincides with the distance travelled by light in time t.* As hypothesised above, the coincidence of values between proper time and spatiotemporal interval, beyond the mathematical devices on the change of sign, is precisely the result of a happy combination of definitions.

8) *PARADOXES (ALLEGED AND TRUE)*.

SR is full of paradoxical phenomena (the rod in the barn, the tiger in the cage.....). Some so-called paradoxes are not paradoxes at all, such as the well-known one of twins, where it is not a matter of two RF's in uniform motion, but of accelerated motion, which is why the absence of symmetry dissolves the false paradox. Experiments on time dilation, actually performed, since the first of its kind conducted in 1971 by J. Hafele and R. Keating ,[128] have confirmed SR's predictions. Most of the 'cases' prove SR in a striking way. The prototype is represented by Einstein's elegant experiment-example on the conductor and the magnet, for which there are two entirely different ways of describing it, depending on whether one or the other is considered to be at rest. If we assume the conductor to be at rest, we have an electric field; if we assume the magnet to be at rest, we have an electromotive force in the conductor. Among the most famous is the very clear one of muons,[129] which are produced in abundance by cosmic rays in the earth's atmosphere: without relativistic effects, the average lifetime of muons would be the same for muons at rest and for those in motion, and the experimental data would be completely inexplicable. Applying the SR equations, we have that *t* measured by the observer in solidarity with RF earth is 7 times longer than the time taken by muons measured in the particle reference system itself (t_0 the muon's own time):

$$\frac{1{,}3 \times 10^{-7\,sec}}{1{,}8 \times 10^{-8\,sec}} \approx 7 \qquad (38)$$

Experimental results on muons can only be explained and predicted with very good accuracy by applying SR. Some so-called paradoxes are, on the contrary, subtle puzzles with traps and pitfalls. True or alleged, they must be analysed. They can, when clarified, constitute a confirmation or a clue to a deeper difficulty. The emergence of paradoxes is often linked to cases of circularity and self-reference, such as Douglas Hofstadter's 'strange loops' in 'Gödel, Escher, Bach' ,[130] but in certain cases they are due to disagreement, real disagreement, on the simultaneity of events and/or reciprocity, whereby each of the IRSs in relative motion measures in the other the shortest lengths and the longest durations, compared to their own, measured with their own rulers and clocks. In perfect symmetry. There is no other way but to rely scrupulously on logical and experimental evidence. The mathematics of SR is relatively simple, but the theoretical mechanism is a real conceptual maze. This is because of the three mutually interacting effects of the transformation equations: time dilation, length contraction and relativity of synchronisation. «Uniform motion is simple, but things are not always simple».[131]

The mechanism of SR, reduced to the simplest scheme, consists, in the typical configuration, with the origins coincident in time t_0, the axes parallel, the directions coincident, by an IRS *S'* , considered at rest, and an IRS *S*, considered in relativistic uniform rectilinear motion with respect to *S'*. The bodies integral with *S'* are, therefore, assumed to be at rest and the bodies integral with *S* in motion at constant velocity $\boldsymbol{v}$ with respect to *S'*. The former have in the reference *S'* length L_0, while the latter will, seen from *S'*, be contracted by the Lorentz factor. One can consider, reciprocally, *S* at rest and *S'* in motion at constant velocity $-\boldsymbol{v}$ with respect to *S*, with completely symmetrical effects in the measurement observations. To express ourselves with Resnick[132]: all these results can be reversed. The phenomena are reciprocal. A's clock goes slow for B, just as B's clock goes slow for A. The metre of A appears contracted to B, in the direction of motion, just as that of B appears to A, contracted in the same way. The factor of shortening of lengths and slowing of times is always $\sqrt{1-\frac{v^2}{c^2}}$ but in the case of lengths we operate a multiplication, in the case of durations a division. The observer in the transformed reference, depending on whether we assume *S'* or *S* at rest, will measure a contracted *l* and a slowed *t* and will also verify that the moving clocks are not synchronised. The contraction of lengths in the direction of motion interacts with the dilation of durations and the relativity of synchronisation. This means that a body, in *S* as in S', has a different clock at each point, and that, in the two RF, time has different rhythms, (clocks in motion or, even, in a gravitational field go slower) and that the relativity of synchronisations (measurements cannot be simultaneous) also occurs. I measure the motion of the moving object with respect to me, combining its variations in a new spacetime line, based on my own time and my state of stillness, temporal and spatial. The three relativistic effects must be combined. It will emerge that the information, on which the calculations must be made, grows very rapidly. And beyond a certain threshold, it is simply not possible to make exact measurements, even in principle. Approximate measurements are, of course, possible. If you then creep in the infinities, all sorts of things come up. The basic question is whether the contraction of lengths and dilation of durations are actual physical processes. Unfortunately, the assessment procedures are very complicated and, as far as the contraction of lengths is concerned, generally beyond practicability. To simplify, we can choose models in which the effects of the dilation of durations and the relativity of synchronisations are minute and can be considered negligible. We lose in rigour, but we can focus on length contraction. The application of the contraction of lengths leads to remarkably enigmatic situations in the following cases, which *cannot be overcome with the correctives from the other two effects*. The theory must ensure that observations from a plurality of RF can be compared in such a way as to be consistent. Instead, one obtains, depending on the case, either consistent results or results that are inconsistent with each other and with the observational data.

a) Let us take, 'drilling where the wood is thin', the case of the rod and the barn. Slightly adapting the standard version (involving a rod and barn), let us imagine two athletes with rods A and B, respectively, of the same length, at an equal distance from barn F, on two parallel axes, on opposite sides. The athletes with rods A and B are both at rest in time t_1. At the next instant t_2, the athlete with rod B starts running with speed $\boldsymbol{v}=\boldsymbol{0}.95\ c$, while A remains at rest. An observer in solidarity with F will see the rod B , in motion towards F, contracted according to the Lorentz equation by a factor of $\frac{\sqrt[2]{1-v^2}}{c^2}$ while he will see the rod A not contracted. The athlete with A, who considers himself to be at rest, will see F non-contracted, while the athlete with B, who is in motion, will see F contracted and will be contracted, seen by F. Therefore, B sees F contracted and can enter and be in F, which in its own reference is non-contracted and therefore can contain B, which in turn is contracted in the reference of F. But A and F are not in relative motion to each other and see each other non-contracted. What is the length of F at the instant t_2 ? That seen by A, non-contracted, or that seen by B, contracted according to the Lorentz transformation? All legitimate, all correct, one answer, because they are different references.

It does not, of course, escape our notice that the distance is different in the two references, that time flows at a different pace for the three observers (the two rods A and B and the barn F each carry their own clocks) and that the clocks in the instant are not kept t_2 synchronisation of the clocks in the instant t_1, but it does not change the conclusion to be drawn at all, because Barn F cannot be an accordion that varies in length in fractions of a second, or at the same instant, for different observers. Each observer speaks of lengths measured at a given instant on their own time axis. They have a different measure of length because they have a different surface of simultaneity. But the experiment implies that one can correctly imagine an instant of touch point or *contact point* between the two RF, in which the length of the barn cannot be two or threefold (the barn's own length, the length seen by B and the length seen by A), unless they are perceived lengths, but without an actual counterpart. Or does nature really behave that way there?

b) Similarly, the model of the cage and the tiger can be analysed. In the reference integral with the cage, it will have l_0 capable of containing the tiger, contracted because it is in motion, which, running at relativistic speed towards the cage, will see it shorter by a factor of $\sqrt{1-v^2/c^2}$. Thus the tiger sees the cage shorter, while the cage sees the tiger shorter. The two detections are both legitimate and correct, except that since the cage has a photocell so that as soon as the tiger was all the way in it would immediately close, only one of the alternatives could be considered real. This is what we have called an instant touch point or *contact point* between the two RF. Either the hunter would capture the tiger or it would happily take the goat out of the cage. Obviously, there are no tigers in nature that run at relativistic speeds and decisive experiments are often unfeasible, as R. Penrose[133] often reminds us. Relativists have their answer ready, based on the relativity of simultaneity (time difference between the snout and the tail), there is no shortage of those who take up the pre-Einstein mechanism of the contraction of bodies (the poor tiger would first be intact in an instant $t_1$then physically compressed and shortened in a time t_2, then whole again in a time t_3) with respect to the ether (Lorentz-FitzGerald physical contraction mechanism), but one only has to have the patience to make the diagram and perform the calculations, to make it manifest that of the two hypotheses only one would occur with all due regard for reciprocity. Recourse to the relativity of simultaneity and synchronisation does not seem sufficient. The SR mechanism can, therefore, be critically examined using its own tools.

c) Even with one of Einstein's classic examples, the platform and the train, it is not difficult to model in a similar way to the previous cases . Let us take a train stopped in front of a platform and two rods, one on the train and the other on the platform, perfectly superimposed, with $l_0 = 1{,}5$. With the train in motion at relativistic speed, the rod in the train reference will have its own length 1.5 , while the rod on the platform will be seen to be shorter, according to the contraction factor. Identical results, in the train reference, if we assume that the train is at rest and the platform moves away from the train at relativistic speed. Let us now assume that the train accelerates to 0.999999 c*;* in the train reference the rod will still have its own length 1.5 , while in the platform reference, i.e. seen from the platform, the rod on the train will tend to zero. Let us now imagine that the train brakes and descends at 100 km/h; in the train reference the rod will be 1.5 long and in the platform reference the difference from 1.5 will be practically irrelevant. The model is obviously not realistic, because there are no trains travelling at relativistic speeds. There are, however, particle accelerators and muons, but l_0 *and* l' are not, at least at present, measurable. Instead, the time dilation effect is detected and measured, in full accordance with SR. The two effects, dilation of durations and contraction of lengths, do not have the same degree of experimental confirmation. Only time measurements are tested and solidly established, and only these appear to possess primary character.

d) Let us imagine, again, an apparently simple experiment (essentially not dissimilar to Ehrenfest's paradox)[134], in which a cross-shaped object, with four arms, rotating around its central axis, travels eastwards with relativistic velocity. The arm moves both perpendicular to the motion of the object and, periodically, in the direction of the motion itself and, consequently, we see it, when in the direction of the motion, shortened by the factor $\sqrt{1-v^2/c^2}$, to manifest, when it moves neither parallel nor perpendicularly, a contraction by a factor between $\frac{1}{\gamma}$ and 1 and reach again l_0 , when it is in a perpendicular position. Would such a periodic contraction/elongation sequence of the rotating arms really occur? If it were a gear in some machine it would cause it to jam or break. Does this happen in nature? The mainstream explanation is not entirely persuasive.

e) Lastly, another Gedankenexperiment, following in the footsteps of the famous '*If I could chase a ray of light*'. Let us imagine a miniature device, like a small rocket, equipped with an apparatus of sensors, capable of measuring, even at great distances, time intervals and spatial lengths, travelling at uniform relativistic velocity, close to *c* , e.g. 99% of the speed of light, on a rectilinear trajectory parallel to a beam of light and in the same direction. The device can, for SR, be assumed to be at rest, either with respect to the light beam or to *n* objects in relative motion, with respect to the device, at very different speeds. What will our device observe? As a first experiment, it will detect that the light beam is travelling at speed c, as predicted in SR, and as a second experiment, again in application of SR, it will observe the positions, time intervals and, therefore, the velocities, dependent on motion, of the *n* objects. The device will consequently detect the effects of time dilation and length contraction according to TL. Any physics student could easily do the calculation for the various objects by setting the necessary parameters. To summarise, the closer the velocity of the objects, which differs from one to the other, approached *c* , the more distances and intervals would tend towards zero, without ever being able to reach it. On the other hand, assuming the other objects to be at rest (as they are integral to their own reference), alternately, there would be a reshuffling of lengths and intervals and it would be our device, observed by several IRSs alternately at rest, to shorten and slow down, until it approached zero. Let us express the same concept in another way: we were sitting in an armchair, observed from the RF of a swarm of cosmic rays, move at a speed close to *c* and our values of length, time and mass will, always in that reference, conform to the TL. This is how the principle of relativity works. Does all this happen in nature? We also seem to detect a bit of unphysical prediction.

Experimental data confirm, so far without denial, the TL predictions as far as time dilation effects are concerned, but for length contraction effects there are, as far as we know, no experimental results except indirect and of doubtful interpretation. Are the lengths ('being' is not equivalent to 'appearing' to the observer) actually shorter and longer, due to their motion and/or the observer's varying motion? Among the experiments, which, albeit indirectly, can be adduced in support of the actual contraction of lengths, see e.g. the discussion by E.M. Purcel[135] on the interaction between moving charges, with different Lorentz contractions between two charge distributions.

In analysing application cases of SR one can run into, as we have shown, intricate and elusive, but theoretically correct, configurations, which force many researchers, in order to make the model work, to resort sometimes to forcings, in some cases to the analogue of the Ptolemaic 'epicycles and deferents', that is to say, to solutions too artificial to be persuasive. If one assumes that the consequences of TLs (at least for the contraction of lengths, on which we have focused our analysis) result in an actual contraction and not only in the observer's perception, one is faced with non-unambiguous experimental data, unverifiable cases, conceptual difficulties and even logical inconsistencies. In contrast to time dilation effects (particles in accelerators, muons, etc.), there is no experimental evidence for the actual contraction of lengths, obviously also because of the difficulties associated with the acceleration of macroscopic bodies to relativistic speeds; although, **in SR's conceptual framework, the contraction of lengths is a logical consequence of time dilation**. «At this time there are not direct tests of lenght contraction, as measuring the lenght of a moving object to the required precision has not been feasible».[136] Is this a serious problem for SR? The answer could be yes.

Given the invariant speed of light, clocks in motion change rhythm, observed from a reference at rest, rulers in motion change length, again observed from a reference at rest, and synchronisation is relative. In the absence of a reference to a system at rest, on which absolute time can be identified, clocks of any IRS are equivalent. There is no privileged RF, they are all equally legitimate. It is clear that no IRS and no property allows absolute motion to be detected. *This is the first rule*: no experiment should be able to tell us whether or not we are in motion. If the conditions are symmetrical, the results must be symmetrical, otherwise the principle of relativity is contradicted. Let us imagine that observers in motion with different velocities proceed to measure from their RF a rod of non-deformable material placed in an inertial RF at rest in solidarity with us. From IRS in motion with respect to us, multiple lengths will be obtained that differ from each other and from the measurement we take. There is no privileged IRS. For a hypothetical IRS at speed *c*, the length of the rod, which is integral with us, would be zero. The contraction of lengths correctly describes the experimental results, although not all of them as we have seen, but it comes at a price if we understand it as an effective contraction. The rod does not have a length independent of its own and the observers' motion. And this should also apply in the absence of observers. It has a multiplicity of possible lengths . There are as many variable lengths, except the invariance of l_0, as many inertial systems, in principle infinite.

Among other things, there is no point in talking about the state of stillness or motion of a single body. It is also observed that the rod (or the fishing rod, in B. Russell's example) will not only have a different length for each RF in motion, but will have a different length in its own RF depending on whether it is placed in the direction of motion or in another direction. As with lengths, also due to the effect of time dilation, there will be as many times as inertial systems in uniform motion but with different velocities. In principle infinite. Unless one accepts that this is a mere mathematical model, one cannot escape the obligation to give a physical interpretation of the effects of spatial contraction. Should a distinction be introduced between observational perceptions and actual physical entities? Are moving objects really shorter or are they only shorter when viewed from another RF at rest than the first? Is it conceivable, by analogy with the Doppler effect, that the observer is measuring a shift in frequency in one case and a contraction in length in the other case, but that this does not mean that the own length, by analogy with the own frequency, has undergone the change detected by our measurements?

Physical theories should be formulated in such a way that they do not necessarily require the existence of observers and, rather, should aim to grasp what is real in the absence of observers. Nature is not at all interested in the observer. A theory that makes physical reality dependent on me observing, or on observation in general, is a problem, and not only for quantum physics, which is frequently reproached, because a theory should not refer to the observer in its postulates, but also for SR, whose author relentlessly tried to corner Bohr on this very point. It is true that SR searches for and defines invariants in changing reference systems, but it is inevitably a theory of the observation of reality.

This is how B. Russel, for example, thought: «Measurements of distances and times do not directly reveal the properties of the things measured, but the relations between the things and the measurer».[137] Spatiotemporal measurements express the relationship between the observer and the thing observed, while the invariant in SR is the interval. Russel[138] added: «The same interval in different regions does not imply exactly the same time» .

9) We will now draw attention to another open question in SR, which concerns the four-dimensional formulation and thus the fundamental concept of spacetime. Space and time, in relation to both TLs and relativistic effects in general, do not have exactly the same behaviour in mathematical formalism. This is despite the fact that the time coordinate is considered equivalent to the space coordinate, through the mechanism of multiplication of duration by c and by $\sqrt{-1}$. We see how this affects the effects of length contraction and time dilation. It is a point on which even in various textbooks, and even in classical texts, it is not uncommon to find arguments that cause confusion and even outright misunderstanding of the theory. The equation of the length observed from another IRS, i.e. seen in the transformed system, calling l_0 the length proper, is:

$$l = l_0 \sqrt{1-\beta^2} \qquad (39)$$

the equation of time, measured by another IRS, seen in the transformed system is:

$$t = \frac{t_0}{\sqrt{1-\beta^2}} \qquad (40)$$

In the first case one multiplies by γ, in the second one divides by γ (analogously to what happens with the mass,

$$m = \frac{m_0}{\sqrt{1-\beta^2}} \qquad (41)$$

where m_0 is the classical mass and m the relativistic mass)\. The above means: proper length $l_0 > l$ (length observed by another IRS) and a clock at rest (i.e. when it marks its own time) faster than the observer. Clocks, on the other hand, slow down when observed from another IRS. One finds, even in very authoritative authors, conflicting interpretations. These are clearly problems of defining terms and everything is made more subtle and complex by the fact that motion is relative and that each IRS can legitimately consider itself to be at rest. For example, Resnick[139] himself writes (p.65) : « The time interval indicated by a clock is longer for an observer for whom the clock is in motion than for another at rest with respect to the clock», which may mean the same thing or even the opposite of what we read on p.62 : «A clock goes at a faster rate when it is at rest with respect to the observer. When it is moving at speed v relative to the observer, its measured rate slows down». To provide clarity, fortunately for us, there are equations, specifically $t = \frac{t_0}{\sqrt{1-\beta^2}}$ and Resnick summarises, now clearly[140] «...a clock moving at constant speed relative to an IRS containing synchronised clocks will be seen to go slower when compared to these clocks. We compare a moving clock with two synchronised stationary clocks».

The different behaviour noted (division or multiplication by γ), from a mathematical point of view does not entail any inconvenience, it could be related to covariant or contravariant behaviour or even to the difference in sign between spatial and temporal coordinates, but what does it mean from a physical point of view? The Lorentz factor γ is a contraction factor, while its reciprocal $\frac{1}{\gamma}$ is an expansion factor. Time dilation, moreover, is an effect that affects not so much the clock instrument, but the passage of time itself. In fact, Δt, the non-proper time interval, is slowed down with respect to proper time τ and therefore clocks run slower in the IRS in motion, seen from the reference at rest. The effect is called the time dilation or retardation of time. The effect of length contraction, on the other hand, cannot affect all spatial dimensions, but only one of the 3 components, that in the direction of motion. But the direction of motion is not necessarily constant, it can vary, even periodically, and even for a body in uniform rectilinear motion (see Chapter III, point 8 d). Can this space/time asymmetry be reconciled with the fact that the time coordinate is treated in the four-dimensional formalism as a fourth spatial dimension? We should add that in SR, imprecision and misunderstandings are not uncommon, even in reference works. E.g. Rindler[141] in the cited work, chap. II, 11, writes about the 'clock paradox': «Since only one of the clocks remained fixed in such a system, the symmetry between them is illusory and the experiment devised simply emphasises the role of these privileged systems». It is quite clear that this is not the case in SR. Further[142] on, one can read: «We derive from this that in physics the speed of light plays the role of an infinite speed in so far as no 'sum' of lesser speeds can ever equal it». It is well known that there is the limiting speed, but also that it is a finite quantity!

10) Restricted relativity, which concerns a particular type of RF, inertial systems, later evolved with GR (1916) into a general theory of arbitrary reference systems, i.e. concerning any RF. In the extraordinary new scientific frontier, a refined and powerful mathematical tool, the absolute differential calculus, was to be used, and is still used today. SR proceeds, on its part, with simpler formalisms. We will now

show the role played in these by approximation. SR is a theory of inertial systems, which by definition have two requirements: straight-line motion and uniform velocity. If one reflects on the concepts of line or one-dimensional continuous space and surface or two-dimensional continuous space, it may be clear that there is nothing in the three-dimensional space of our experience that can have strictly one or two dimensions. Moreover, another dimension, time, is needed to describe most phenomena. Subjecting the concepts of uniform rectilinear motion and inertial systems to the same kind of analysis to which we subject geometric objects leads to the conclusion that they are not physical realities, but mathematical entities. It goes without saying that SR procedures are only valid and effective when the curvature of spacetime is negligible. We have no way of measuring time and space intervals in a strictly objective manner. Real motion, on closer inspection, is all non-rectilinear and non-uniform and its velocity determinable at an instant only by the slope of the tangent to the curve at that instant. Observers, likewise, would all be accelerated or subject to gravitational fields. It is Einstein himself who writes, in *'The Evolution of Physics*[143]: «However, uniform motion can never take place», due to external forces, which cannot be totally eliminated. Similarly, he wonders in the part 3°, as we have pointed out, whether an inertial system exists in reality. This is an idealisation, like geometric entities. Uniform motion would require the traversing of absolutely equal spaces in absolutely equal times. Do we have infinitely precise measurements of rectilinear character and velocity? In curved space nothing moves in a straight line, except locally, by *convention.* For SR, spacetime is a flat sheet. The theory's own formidable invariants are approximate quantities. Simultaneity and synchronisation appear, in principle, unattainable. To the question of whether, in the end, we have not replaced one absolute with another, one must plausibly answer that we have done so of necessity and by means of *assumptions* and *conventions*, which turn out to work with extraordinary effectiveness.

One can investigate, in another respect, how SR functions with approximate characteristics in the description of certain classes of phenomena. It is not only a matter of the fact that every measuring apparatus always provides approximate values. Let us take the measurement of lengths and distances in a very simple IRS, which we shall call A, e.g. the usual railway platform, observed in its own RF at rest. We cannot, of course, know whether A is stationary or in motion, but no problem, because all effects must be symmetrical; however we are allowed to assume it to be at rest. We measure the length of the stationary train, relative to A, along the platform. The measurement of the ends of the train must, as we know, be simultaneous and this has as a condition that the light emanating from the ends arrives at the same instant at the midpoint of the train. It is a very simple operation, but if we propose not a FAPP measurement, but an exact measurement, we immediately realise that we must make use of *assumptions* and *conventions*, which are necessary but cannot be supported by logical demonstrations or exact and objective empirical data. The first mode, i.e. the procedure to achieve simultaneity, presupposes simultaneity itself or, if you prefer, a synchronisation scheme, while the second requires very exact measuring instruments, one would say of infinite precision. We also have, of course, the formula

$$l_0 = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \qquad (42)$$

where $\Delta x = x_2 - x_1$, etc. I have a different opinion, for this reason, with respect to Y. Z. Zhang,[144] when he writes: «The lenght measurement of a rest rod is simple», because this measurement hides big assumptions. The same thing happens if we want to measure the length of the train whistle, even more so if it is emitted by a locomotive far away from us. If we then apply GR, we repeat, each point in spacetime has a different ruler and a different clock. Even the measurement of a body observed in its reference of stillness is not as simple as one would think at first glance. Exactness would, strictly speaking, require that the measurement be made not with reference to a segment, but to a point. If then the body, even legitimately assumed to be at rest, were in motion, as is the case in nature, one can easily see that the matter becomes not a little more complicated. Absolute exactness appears unattainable, both in the measuring operations and in the determination of the initial conditions. Even in idealised models of SR, which constitute the simplest possible situations, measurements are approximated, perhaps wonderfully and impressively. Turning to real systems, the approximations move further and further away from exactitude, while remaining, for the most part, entirely satisfactory. Not dissimilar to this argument is that of Disalle[145]: «GR implies, with respect to SR, that what is true in an inertial system is only true locally, which is equivalent to stating that in reality there are no inertial systems». But there is a more authoritative author who thinks this, as we have seen, and he goes by the name of Einstein. All that said, there is no doubt that the relativistic approximation comes closer to reality than the classical approximation. After all, to use Feynman's expression «Physical laws are all some kind of approximation»[146], or, also quoting a saying by von Neumann[147]: truth is too complicated to allow anything but approximation. We have just shown that uniform rectilinear motion, in a curved spacetime, assuming it could exist in reality, would have to be very circumscribed indeed, and the motion could generally only be curved. The gravitational field is described by a curved spacetime variety. The inertial reference system is, in short, a useful mathematical abstraction. Minkowski's four-dimensional geometry consists of a pseudo-Euclidean space with positive undefined distance and zero curvature and does not describe reality exactly, as it is a strictly local, point-like and flat view, devoid of acting forces. For each observer, each clock marks a local time and each ruler will have a different measurement for each observer moving at a different speed. The world of SR is an abstract world in which all mass and energy is removed, an empty world. Einstein[148], in his *Scientific Autobiography*, expresses this state as «…the simplest of all imaginable physical situations». A world that is, in a sense, fictitious. The discussion of approximation in SR should be placed within this framework. Of course, in using the term 'fictitious', it does not escape one's notice that immediately the opposite could appear, since we are habitually dealing with an almost flat spacetime, with a curvature so small that it can be neglected FAPP. As Luciano Rezzolla[149] happily writes : the famous sheet, the most popular depiction of gravity, is difficult to curve.

It might seem that nothing changes between a situation in which the measurement is exact and a situation in which the measurement is approximate but works satisfactorily *for all practical purposes.* The subtle difference, not for practical purposes, is between measurements that can be exact in principle but are inevitably approximate in practice and measurements that cannot be exact in principle. Deviations may or may not be negligible and insignificant, but when the approximation involves measures of space and time, i.e. the scope of SR, eliminating the approximation would be tantamount to calculating with speed ∞ in contradiction with SR itself, which prohibits a signal from propagating instantaneously. «Equations of this kind are very popular also in relativistic settings (especially in relativistic astrophysics) and are employed in theoretical models and numerical simulations».[150]

Finally, three critical points are worth mentioning:

* The photon's rest mass: photons have no rest mass. According to the theory of relativity there is no essential difference between mass and energy, and mass-energy can be expressed in units of energy or equivalently in units of mass, and their units differ by a factor of c^2. It follows that particles of zero mass at rest can be assigned a mass equivalent to their energy. Energy has mass and mass represents energy. The concept of mass at rest is considered by many scientists to be outdated and even somewhat cumbersome. Light is deflected by the gravitational field. Photons have kinetic energy, but zero rest mass. See also Resnick[151]: « The term zero rest mass is somewhat equivocal in that it is impossible to find an RF in which photons are at rest». The concept of rest mass, Susskind[152] notes, is anachronistic. 'Mass' now means what 'rest mass' once meant. Energy at rest is called mass. Einstein[153] himself clarifies: «The inert mass of a closed system coincides with its energy, and mass is thus eliminated as an independent concept». Photons are never at rest, there is no RF in which photons are stationary and in a vacuum they can only propagate at speed c. Photons in motion, at speed *c*, always possess mass, in the form of kinetic energy. The mass of a body, for Einstein, is a measure of its energy content. The mass associated with energy, in its various forms, possesses the properties attributed to mass and this renders problematic the definition of photons as massless and certain consequences that are drawn in SR's corollaries, e.g. that for zero mass, time is zero. Indeed, if one refers to the relationship between mass and time one is calling into question, among the properties of mass, inertia, which depends on the energy content. The concept of m_0 poses problems in general, apart from the case of photons, when it is referred to the microworld, in which the absence of motion generally does not take place. Within protons and neutrons, quarks and gluons move very fast! Or consider, e.g., the impossibility of inelastic collision, since the contribution to mass of kinetic energy must be taken into account.

* Thermodynamics offers us, with its description of processes in time, an idea that should fit with relativity. For SR photons travel at speed c and at that speed time is zero, on the other hand they undergo wavelength and frequency variations in their motion. Such processes, considered in their own RF, would occur in a $\tau = 0$. Any process, in its own RF, requires time to flow, for any change requires time. Can reality change in zero time? For photons, time does not elapse, but they do change. In the reference of a clock in motion at speed c, therefore with time equal zero, would change be permissible? For it cannot be $\tau = 0$.

* A rather obvious contradiction is found between SR and Quantum Theory on the energy-time complementarity applied to a beam of photons. If $t = 0$, $dE\ dt$ will not be greater than cut acca, as the uncertainty principle dictates.

It is not advisable to consider problems solved as long as there are some questions that are not really solved. Resnick[154] writes that «…relativity is absolute and simple», but I think that nothing should be taken as too obvious. It sometimes happens that uncomfortable objections or observations are swept under the carpet. Einstein, on the contrary, wrote in a letter to his friend Besso[155]: «After 50 years of conscious speculation, I have still not come one inch closer to the answer to the question "what are light quanta". Today in truth any beggar thinks he knows, but he deceives himself».

On some important issues, there remain not insignificant grey areas. The critical observations made here are certainly not fatal, but they are such as to raise reasonable questions about the consistency between theory and experimental reality. Things that do not add up are not to be taken as annoyances. They are stimuli.

Observations and experiments in progress and planned for the years to come, greater computing power, new-generation instrumentation and tests, e.g. with modern electronic and photonic technologies, hitherto unthinkable frontiers of sensitivity to physical effects, third-generation precision cosmology, will be able to address many open questions experimentally and try to clarify things better. Certainly, new technologies and methodologies will enable us to describe and simulate spacetime relations much better.

Several points, which are solid for us, are in fact called into question by the experiments in progress or in the pipeline. New data may confirm the symmetries of SR, but it cannot be excluded that some of the current experimental data may become obsolete and the theory may be 'displaced' on some points, opening the way for necessary revisions.

Quantum entanglement, e.g., has revealed an incompatibility between special relativity and quantum physics. Formidable experimental tests have been and are being carried out on the basis of Bell's inequalities, which allow certain physical and philosophical theories to be put to the test. It has been happily referred, by many researchers, to as 'experimental metaphysics'. Bell's theorem consists in being able to experimentally test the compatibility of SR and QT. The denial of the possibility of detecting simultaneous effects in the case of entanglement is not, as yet, experimentally confirmed.

The ascertainment or non-confirmation of simultaneity at a distance has a diriment value on the relationship between relativity and quantum theory. The former denies the possibility, the latter demands it (entanglement and, but this is a different matter, wave function collapse). Depending on the experimental results on entanglement, one must either modify special relativity or modify quantum mechanics. Or, both. Shimony[156] for example suggest the conjecture that there is no space between two entangled particles.

Let us close the Chapter III with an aspect of the relationship between SR and GR. Operating with SR and Minkowski's flat spacetime is clearly very simple and convenient. Much less simple is to operate in the curved spacetime, which is described by a 4-dimensional curved variety, on which one defines the metric, operating with scalar products on the induced tangent space, where the tangent space at each point is a Minkowski space. As is well known, the most powerful mathematical tools are provided by tensor algebra and tensor analysis. The tensor field describes invariances in a more general way than vector fields. In fact, in a nutshell, if an equation is valid in SR, when the equation has a tensorial character, when it is therefore independent of the choice of a basis, by applying the transformation laws, it is also correct according to GR, i.e. in a curved spacetime. The rule is simple: one starts from valid equations in SR, works through $\eta_{\mu\nu}$, to the metric tensor $g_{\mu\nu}$, which allows us to rigorously define concepts such as distance, angle, geodesics, curvature and so on, and from ordinary derivatives to covariant derivatives and we obtain equations that are also valid for GR, i.e. in the presence of gravity. If we have a valid covariant tensor equation in SR, it remains valid in GR. In this way, generalisation from IRS to RF of any kind is possible, also because the tensor equations are formally invariant. One can thus move freely from one to the other and the same mathematics can describe both worlds, flat and curved space (on the relationship between SR and tensor analysis see also the article '*On Spacetime*').[157] The central point is that tensors make it possible to distinguish between, let us say intrinsic, geometric or physical properties and those dependent on coordinates. The procedures based on the tensorial transformation rules allow for remarkable results, precisely because of the invariant character of the tensorial equations, but obviously they cannot resolve, when there are any, the conceptual difficulties, which are the object of the present analysis, of the original equations, from which the tensorial transformation rules are applied.

Chapter IV : CONCLUSIONS

The article focuses on kinematics, first and foremost, and dynamics, because the rest, on closer inspection, is derivative, but the analysis will have to be extended to electricity and magnetism for completeness. We can, however, at this point extract and put in order the first results.

In Chapter I:

We have shown that the relativistic equation of addition of velocities is a mere tautology and by extending the examination to the invariance of *c* and the concept of limit velocity we have shown that SR has, in essential parts, observational-experimental, not logical-deductive foundations. The consequence is that the domain of applicability is not universal, but rather confined to the empirical field considered. What is grounded in experience can be corrected or refuted by experience and is only ascertained for facts that do not fall outside its domain of applicability. By summarising the theoretical and experimental history of SR, we have been able to ascertain that SR is not, to date, contradicted by experiments and is, therefore, validated, as the vast majority of the scientific community consequently believes. The theory, however, can only be generalised, conjecturally, to phenomena outside its domain of applicability. Anyway, these limitations are not always adequately taken into account by researchers of astrophysics and cosmology.

On the simultaneity and synchronisation of clocks, we have come to three conclusions:

a) simultaneity and synchronisation are not possible without a set of assumptions and conventions; b) point a) above also affects the isotropy of light, the homogeneity of space and the homogeneity and isotropy of time;

c) with regard to spatio-temporal relations, it is necessary to investigate the combination SR + GR more closely;[158] the application of this, in a logically and mathematically rigorous manner, would lead e.g. to a very powerful statement: only two clocks in a point system (= a single clock) can have exactly the same temporal rhythm.

In Chapter II:

The relationship between theory and experience (and between deduction and induction) in Einstein's thought was examined. It is evident from the texts that his positions do not remain unchanged and that significantly different positions are present at different times, but the compass is constant, for which deductions, hypotheses and conjectures must not be refuted and rather must be validated by empirical evidence. Einstein always held that "only experience can decide" and that a single experiment can sweep away an entire theory. The corollary that follows from this is that capital points of SR, such as the speed of light, the upper limit of speed, the effects of time dilation and contraction of lengths, etc., are not just coherences in a logical-mathematical construction, but are **objects of experimental physics.**

In Chapter III :

Some conceptual difficulties of the theory were identified and discussed, and several open questions in SR were indicated, and more than three critical points were added as examples.

By listing, it was shown:

* The concept of an inertial reference system is not a description of nature, it is, rather, a useful idealisation;

* The determination of quantities in a reference of stillness, i.e. l_0 , t_0, and consequently m_0, does not evade the need to resort to *assumptions* and *conventions*, as is the case for measurements in any RF; furthermore, a question emerges: does the reference at rest in SR constitute a preferred reference?; this would immediately be followed by another derived question: in SR, should not all references be equally legitimate and all on the same plane?

* Instantaneousness in entanglement and in the collapse of the wave function are not compatible with SR; there is work for experimentalists, as far as entanglement is concerned, whereas the collapse of the wave function is a theoretical hypothesis outside the experimental sphere, at least for now;

* The spatiotemporal interval is one of the most subtle and difficult questions, but (Chapter III, point 7) it is shown that it is not a definite, i.e. closed question; it is the relationship between spatiotemporal interval and proper time the result of a happy combination of definitions.

* An examination of the so-called 'paradoxes' (excluding the alleged ones) shows that the accounts that do not add up are not to be swept under the carpet, but, especially with regard to contracted lengths, it does seem that we need to think more deeply;

* The different behaviour in the mathematical formalism for space and time gives rise to a complex discussion on the four-dimensional formulation of relativity and generally proposes non-negligible problems for the four-dimensional theoretical construction;

* Exactness and approximation in the application of theory: we have efficient and convenient calculation tools, which work for ordinary purposes; we can accomplish a lot of things with them; they also work for the usual needs of science, but exactness and axiomatic rigour do not seem attainable;

* In the cited article *On Spacetime*, being published, to which reference is made, four-dimensional formalism and the foundations of the theory of spacetime, i.e. the four-dimensional structure of reality, are discussed.

We have arrived at the crucial point. How should we interpret SR? It is an abundantly proven theory. Not only proven, even working and majestic, to use F. Wilczek's[159] expression, but, nonetheless, logically not evident in some fundamental points and mathematically only partially rigorous (as shown in Chapter III, point 10, on the role of approximation), obviously within the limits of accuracy of observations and measurements. The limits have no practical effects, in everyday life or even in experimental scientific activities, but can have important theoretical consequences on both GR and QM, two pillars of contemporary physics. Please refer to the article *On*

Spacetime, among other points, for the issue, which cannot fail to strike, of the absence in QM of an operator for time.

With SR, supplemented by GR, we have the clock and the metre. We also have a set of hypotheses about spacetime that have not so far been contradicted by experience. With the aforementioned characteristics: a) our entire arrangement of spacetime is based on a quantity of an experimental nature; b) we cannot know either the objective length or duration of event or object; c) the world of the IRS is an abstract entity. SR gives us, certainly, two experimentally based data: 1) the invariance of *c*; 2) the invariance of spacetime separation. A third type of invariant, often not included among these, are length, time and mass, measured in the reference of stillness. It also gives us many other results: just to give an example, a falling grave is sliding along the time curve; the 'down' is the direction in which time slows down (Musser[160]) and so many other important results.

Finally, two relevant issues:

A) One aspect, often overlooked, is that many discussions in philosophy and physics in this field of research are rendered unclear by the confusion between time and the measurement of time and between space and the measurement of space. Time is habitually intertwined and superimposed with the measurement of durations and space with the measurement of distances. These are distinct concepts. SR, if we talk about theory and not commentary, deals with the measurements of space and time. The reasoning, repeatedly expounded by Einstein, which, because of the two postulates, rules out the existence of absolute space and time, implies that we observers only have access to measurements, but does not necessarily imply that what is exclusively real are the observers' measurements. Are time and space nothing but the measure of time and space? It is not a question of imagining untested or untestable hypotheses, but simply of not assuming a priori measurement as exhaustive of the whole. Much has been written and is written about space and time, but in fact not much is known about them. There is a great variety of opinions, ranging from considering them as some form of matter to simple tools for coordinating events. Some methodological criteria, however, can reasonably be assumed. It is plausible to assume that time passes in the absence of someone observing or measuring it and that space does not disappear in the absence of someone observing or measuring it. In the certainly possible scenario of a continuation of the world without us and without other intelligent beings, space would continue to exist and time would continue to flow. Time and space can therefore hardly be thought of as dependent on the observer. Would this point of view lead us back to an absolute, pre-relativistic conception of time and space? Would time again be absolute and equal in every reference system? Not at all. It would probably be in accordance with SR's predictions in the various RF. There remains the conundrum of how things would be in the absence of observers in the broadest sense and thus of reference systems. If space and time do not depend on the observer and his measuring operations, it would in a sense mean that they are absolute (the Latin absolutus), but the fact remains that all our experience of them, i.e. observations and measurements, is relative.

   It is well known, beyond the researchers in the field, that there has been a passionate dispute for decades between two titans of physics and also of philosophy, Einstein and Bohr. The former always held two points firm: the perspective of realism and the notion that physics has the real as its object, the latter always opposed the notion that we never interact directly with nature, so that 'objects', both classical and quantum, are mental constructions of our own, to try to tap into the real. Measurements of time and space require and depend on the observer. Claiming that time and space depend on the motion of the observer only makes sense if we refer to the measurements, not to time and space as such. In the second case, we would end up, head and foot, in a form of naive idealism. There is, therefore, nothing pre-relativistic or metaphysical in the viewpoint put forward. I therefore radically disagree with the statement by the 2022 Nobel Prize winner for physics, A. Zeilinger[161] (whose extraordinary experimental results we all greatly admire) when he says: «The idea of a reality pre-existing observation and independent of it is losing ground. It may be useful to leave this concept behind... The concept of an unobservable and unreachable reality is meaningless. Do we have any evidence of a reality that exists independently of observation?» . I find this disarming position the result of naïve idealism. It is a well-known fact that the integral reduction of knowledge to observation leads unassailably, with an unrefutable logical sequence to a solipsistic point of view, which in turn has the minor drawback of clashing with the entire experience of mankind.

B) Lastly, a specifically epistemological aspect: bearing in mind the debate on the completeness or otherwise of physical theories, we must ask ourselves whether relativity constitutes a complete description of spacetime relations. What are the consequences if we apply to SR the criterion of completeness that Einstein demanded of QM in his dispute with Bohr?
His criticism of QT is that it gives an incomplete description of nature. In 1950, he called QT 'a naive theory' (quoted by A. Pais)[162]. Is special relativity a complete theory? There are good reasons, also developed in this work, to claim that SR is, using Einstein's own criteria, an incomplete theory, however correct and proven. « Physics, - he writes - is an attempt to conceptually grasp reality as we conceive it independently of being observed. In this sense one speaks of 'physical reality' »[163] . Although SR, beyond the frequently and seriously misinterpreted term relativity, constitutes a theory of invariants for inertial observers, later extended with GR to arbitrary observers, it is clear, however, that it is a theory of the observation of reality. Indeed, relativistic effects (contraction lengths, dilation durations, relativity synchronisation) necessarily postulate the observer. Nor would the matter change one iota by replacing, following H. Reichenbach' s[164] suggestion, observers with the 'plurality of conventional systems'. SR is in essence a theory of the observation of reality. It does not seem tenable that SR constitutes a complete description of nature, as far as spatiotemporal relations are concerned, according to the expression used critically by Einstein, regarding QM, in the famous talk with Heisenberg in 1926 in Berlin.[165]
It is plausible to think that we are missing something, and something big. SR constituted «…only the first step in a necessary development»[166] (Einstein in *Scientific Autobiography*) and is, until now, with the developments achieved with

general relativity, not having been refuted by experience, the best 'effective theory' of spacetime relations. No less, no more.

Perhaps it is not excessive to think that we have recently emerged, at least as far as fundamental concepts (mass, energy, space, time) are concerned, from a sort of prehistory of scientific knowledge. We still know very little of what it is perhaps possible to know and we are far, in particular, from a satisfactory theoretical understanding of spatial and temporal phenomena.

There are also very authoritative physicists who argue that space, time and spacetime are not necessary at all to describe the world. I mention among all C. Rovelli.[166]

Let us take one of the most familiar, highly confirmatory examples. In order to function, the GPS system needs Euclidean geometry, the invariance of the speed of light and Lorentz transformations, i.e. the theory of special relativity, in addition, Newtonian mechanics, with the corrections required by GR, QM-based atomic clocks and I don't know what else. The result is that GPS with all this network of theories works perfectly! It happens for the theory of relativity, as for quantum physics, that the theoretical and computational tools are very effective in describing, predicting phenomena and realising formidable technologies, even though we often cannot explain why they work in a certain way. We have very powerful theories, «…while we are still debating what it all really means» (T. Rudolph)[167]. There is also no shortage of those (the proponents of so-called 'instrumentalism') who think that theories are not descriptions of reality, but merely mathematical tools useful for doing the maths and representing and predicting observable phenomena.

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- The following abbreviations have been adopted:

  IRS = Inertial Reference System

  RF = Reference Frame

  SR = Special Relativity

  GR = General Relativity

  QM = Quantum Mechanics

  QFT = Quantum Field Theory

  QT = Quantum Theory

  LT = Lorentz Transformations

  LI =Lorentz Invariance

  LIV = Lorentz Invariance Violation

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