Equation of Prolate Spheroid in Spherical Polar Coordinate System

Abolfazl Soltani *

Department of Physics, University of Birjand, Birjand, Iran

Email*: soltani.a.physics@gmail.com*

Abstract

Here we obtain the equation of Prolate Spheroid in Spherical Polar Coordinate system.

Keywords: Prolate spheroid, Ellipsoid, Oblate spheroid

Prolate Spheroid Equation

As you know, a prolate spheroid with scales $a = a_0 > (c_0 = b_0)$ is obtained from the whole rotation of an ellipse with semi-major axis a_0 and semi minor axis b_0 around the a_0 axis. Now we want to obtain the equation of a prolate spheroid in Spherical Polar Coordinate system. Consider Fig. 1. In this figure, a prolate spheroid will draw from the complete rotation of the blue ellipse with scales a_0 and b_0 around the x-axis.

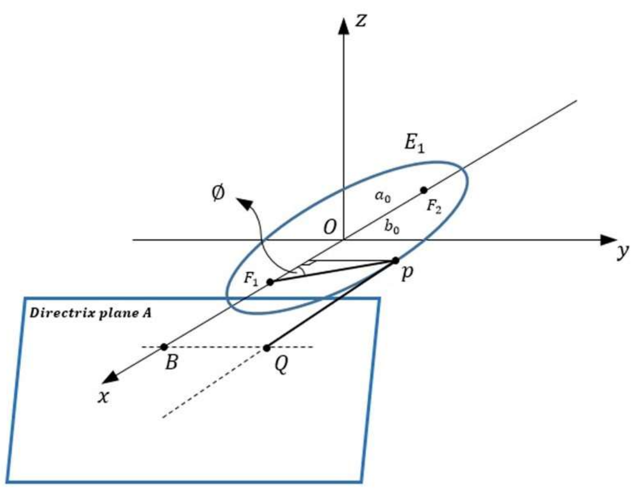

Fig. 1. As shown in the figure, the blue ellipse is on the horizontal plane xy. From the rotation of the blue ellipse around the x-axis, a prolate spheroid will build. The plane A is located at a vertical distance $\frac{a}{\epsilon}$ from the origin O and is perpendicular to the x axis and we call it the *spheroid Directrix plane*. Where ϵ is eccentricity of ellipse.

In the ellipse in Fig. 1 we have [1][2]:

$$\frac{F_1P}{QP} = \frac{r}{d + rSin\theta Cos\emptyset} = \epsilon_0 \tag{1}$$

Where ϵ_0 is eccentricity of ellipse. In this equation, QP is the vertical distance from point P to the *spheroid directrix plane*, namely plane A. This plane is perpendicular to the x-axis and its vertical distance to point O is equal to a_0 / ϵ_0. d is the distance between F_1 and B (point B is on the plane A) and we have: $d = \frac{a_0}{\epsilon_0} - \epsilon_0 a_0$. Equation 1 is valid for all the points P on the surface of the prolate spheroid, due to we can build a prolate spheroid by rotation of the ellipse in Fig. 1 around x-axis. In equation 1, $rsin\theta cos\emptyset$ is the shadow of F_1P line on the x axis and

https://doi.org/10.33774/coe-2022-3rz4n-v4 **ORCID:** https://orcid.org/0000-0002-0482-2584 Content not peer-reviewed by Cambridge University Press. **License:** CC BY 4.0

is written in Spherical Polar Coordinate system $(r.\theta.\emptyset)$ based on Fig. 2, because the spheroid is a three-dimensional object. $d = \frac{a_0}{\epsilon_0} - \epsilon_0 a_0$ and therefore, from 1 we have:

$$r - r\epsilon_0 Sin\theta Cos\emptyset = a_0\left(1 - \epsilon_0^2\right) \quad (2)$$

This is equation of prolate spheroid in spherical polar coordinate system when the center of coordinate system is on F_1 in Fig. 1.

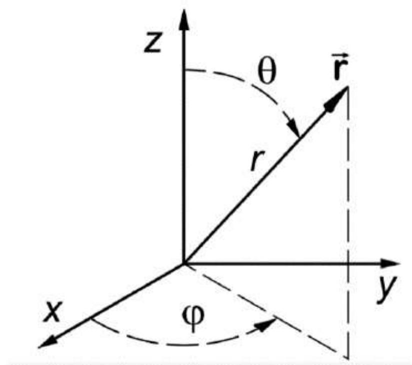

Fig. 2. Spherical Polar Coordinate system

Conclusion

In this article we used from the concept of *Directrix Plane of a prolate spheroid* to obtain the equation of prolate spheroid in Spherical Polar Coordinate system. It's clear the concept of *Directrix Plane* is useable for ellipsoid and subsequently oblate spheroid. *Directrix Plane* seems to be entering Geometry and Mathematics for the first time.

References:

[1]. Silverman, R. *Calculus with Analytic Geometry* (Prentice-Hall, Inc. New Jersey. 1985), pp. 628-641
[2]. Johnson, R. *Calculus with Analytic Geometry* (Allyn and Bacon. Inc. Boston, ed. 4), pp. 536-539