Chapter 2: Two-Dimensional, Steady-State Conduction

Section 2.1: Shape Factors

2.1-1 (2-1 in text) Figure P2.1-1 illustrates two tubes that are buried in the ground behind your house that transfer water to and from a wood burner. The left hand tube carries hot water from the burner back to your house at $T_{w,h}$ = 135°F while the right hand tube carries cold water from your house back to the burner at $T_{w,c}$ = 70°F. Both tubes have outer diameter D_o = 0.75 inch and thickness th = 0.065 inch. The conductivity of the tubing material is k_t = 0.22 W/m-K. The heat transfer coefficient between the water and the tube internal surface (in both tubes) is $\overline{h}_w$ = 250 W/m^2-K. The center to center distance between the tubes is w = 1.25 inch and the length of the tubes is L = 20 ft (into the page). The tubes are buried in soil that has conductivity k_s = 0.30 W/m-K.

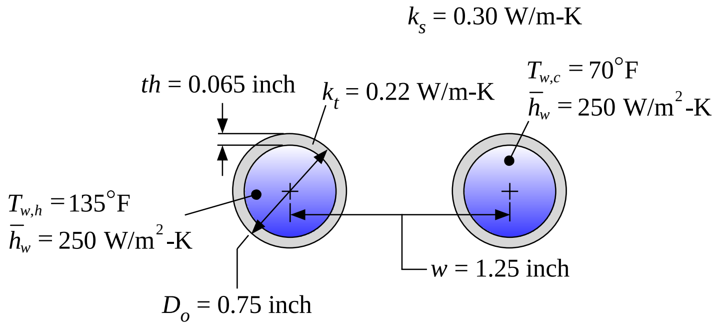

Figure P2.1-1: Tubes buried in soil.

a.) Estimate the heat transfer from the hot water to the cold water due to their proximity to one another.
b.) To do part (a) you should have needed to determine a shape factor; calculate an approximate value of the shape factor and compare it to the accepted value.
c.) Plot the rate of heat transfer from the hot water to the cold water as a function of the center to center distance between the tubes.

2.1-2 Currently, the low-pressure steam exhausted from a steam turbine at the power plant is condensed by heat transfer to cooling water. An alternative that has been proposed is to transport the steam via an underground pipe to a large building complex and use the steam for space heating. You have been asked to evaluate the feasibility of this proposal. The building complex is located 0.2 miles from the power plant. The pipe is made of uninsulated PVC (thermal conductivity of 0.19 W/m-K) with an inner diameter of 8.33 in and wall thickness 0.148 in. The pipe will be buried underground at a depth of 4 ft in soil that has an estimated thermal conductivity of 0.5 W/m-K. The steam leaves the power plant at 6.5 lb_m/min, 8 psia with a 95% quality. The outdoor temperature is 5°F. Condensate is returned to the power plant in a separate pipe as, approximately, saturated liquid at 8 psia.

a.) Neglecting the inevitable pressure loss, estimate the state of the steam that is provided to the building complex.

b.) Are the thermal losses experienced in the underground pipe transport process significant in your opinion? Do you recommend insulating this pipe?

c.) Provide a sanity check on the shape factor that you used to solve this problem.

2.1-3 (2-2 in text) A solar electric generation system (SEGS) employs molten salt as both the energy transport and storage fluid. The molten salt is heated to 500°C and stored in a buried semi-spherical tank. The top (flat) surface of the tank is at ground level. The diameter of the tank before insulation is applied 14 m. The outside surfaces of the tank are insulated with 0.30 m thick fiberglass having a thermal conductivity of 0.035 W/m-K. Sand having a thermal conductivity of 0.27 W/m-K surrounds the tank, except on its top surface. Estimate the rate of heat loss from this storage unit to the 25°C surroundings.

2.1-4 A square extrusion is L = 1 m long and has outer dimension W = 3 cm. There is a D = 1 cm diameter hole aligned with the center of the extrusion. The material has conductivity k = 0.5 W/m-K. The external surface of the extrusion is exposed to air at T_a = 20ºC with heat transfer coefficient $\bar{h}_a$ = 50 W/m^2-K. The inner surface of the extrusion is exposed to water at T_w = 80 ºC with heat transfer coefficient $\bar{h}_w$ = 150 W/m^2-K.

a.) Determine the rate of heat transfer between the water and the air.

b.) Carry out a sanity check on the value of the shape factor that you used in (a).

2.1-5 A pipe carrying water for a ground source heat pump is buried horizontally in soil with conductivity k = 0.4 W/m-K. The center of the pipe is W = 6 ft below the surface of the ground. The pipe has inner diameter D_i = 1.5 inch and outer diameter D_o = 2 inch. The pipe is made of material with conductivity k_p = 1.5 W/m-K. The water flowing through the pipe has temperature T_w = 35ºF with heat transfer coefficient $\overline{h}_w$ = 200 W/m-K. The temperature of the surface of the soil is T_s = 0ºF.

a.) Determine the rate of heat transfer between the water and the air per unit length of pipe.

b.) Plot the heat transfer as a function of the depth of the pipe.

c.) Carry out a sanity check on the value of the shape factor that you used in (a).

Section 2.2: Separation of Variables Solutions

2.2-1 You are evaluating a technique for controlling the properties of welded joints by using aggressive liquid cooling. Figure P2.2-1 illustrates a cut-away view of two plates that are being welded together. Both edges of the plate are clamped and effectively held at temperatures T_s = 25°C. The top of the plate is exposed to a heat flux that varies with position x, measured from joint, according to: $\dot{q}''_m(x) = \dot{q}''_j \exp(-x/L_j)$ where $\dot{q}''_j$ =1x10^6 W/m^2 is the maximum heat flux (at the joint, x = 0) and L_j = 2.0 cm is a measure of the extent of the heat flux. The back side of the plates are exposed to aggressive liquid cooling by a jet of fluid at T_f = -35°C with $\overline{h}$ = 5000 W/m^2-K. A half-symmetry model of the problem is shown in Figure P2.2-1. The thickness of the plate is b = 3.5 cm and the width of a single plate is W = 8.5 cm. You may assume that the welding process is steady-state and 2-D. You may neglect convection from the top of the plate. The conductivity of the plate material is k = 38 W/m-K.

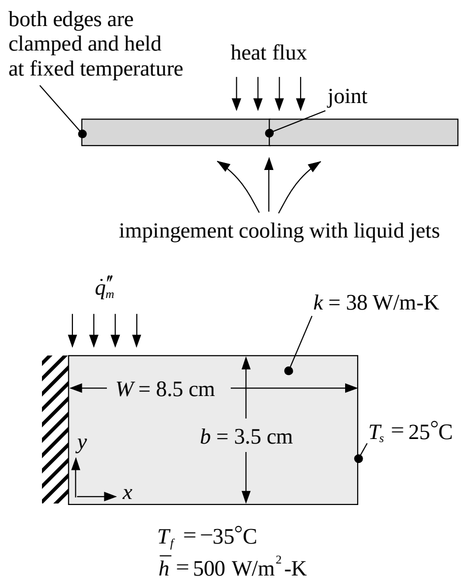

Figure P2-2-1: Welding process and half-symmetry model of the welding process.

a.) Develop a separation of variables solution to the problem. Implement the solution in EES and prepare a plot of the temperature as a function of x at y = 0, 1.0, 2.0, 3.0, and 3.5 cm.

b.) Prepare a contour plot of the temperature distribution.

2.2-2 Figure P2.2-2 illustrates a thin plate that is exposed to air on upper and lower surfaces. The heat transfer coefficient between the top and bottom surfaces is $\bar{h}$ and the air temperature is T_f. The thickness of the plate is *th* and its width and height are *a* and *b*, respectively. The conductivity of the plate is *k*. The top edge is fixed at a uniform temperature, T_1. The right edge is fixed at a different, uniform temperature, T_2. The left edge of the plate is insulated. The bottom edge of the plate is exposed to a heat flux, $\dot{q}''$. This problem should be done on paper.

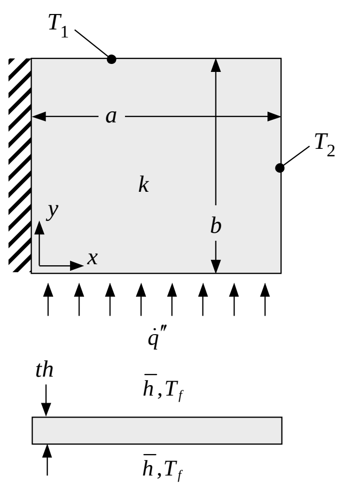

Figure P.2.2-2: Thin plate exposed to air.

a.) The temperature distribution within the plate can be considered 2-D (i.e., temperature variations in the *z*-direction can be neglected) if the plate is thin and conductive. How would you determine if this approximation is valid?

b.) Derive the partial differential equation and boundary conditions that would need to be solved in order to obtain an analytical solution to this problem.

2.2-3 (2-3 in text) You are the engineer responsible for a simple device that is used to measure heat transfer coefficient as a function of position within a tank of liquid (Figure P2.2-3). The heat transfer coefficient can be correlated against vapor quality, fluid composition, and other useful quantities. The measurement device is composed of many thin plates of low conductivity material that are interspersed with large, copper interconnects. Heater bars run along both edges of the thin plates. The heater bars are insulated and can only transfer energy to the plate; the heater bars are conductive and can therefore be assumed to come to a uniform temperature as a current is applied. This uniform temperature is assumed to be applied to the top and bottom edges of the plates. The copper interconnects are thermally well-connected to the fluid; therefore, the temperature of the left and right edges of each plate are equal to the fluid temperature. This is convenient because it isolates the effect of adjacent plates from one another which allows each plate to measure the local heat transfer coefficient. Both surfaces of the plate are exposed to the fluid temperature via a heat transfer coefficient. It is possible to infer the heat transfer coefficient by measuring heat transfer required to elevate the heater bar temperature a specified temperature above the fluid temperature.

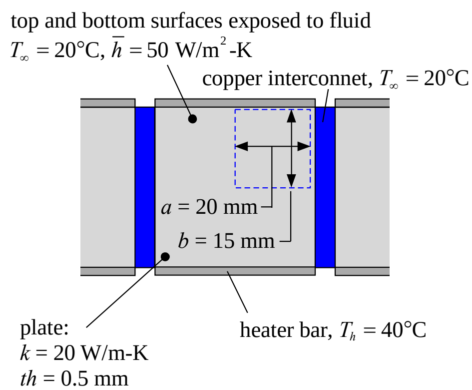

Figure P2.2-3: Device to measure heat transfer coefficient as a function of position.

The nominal design of an individual heater plate utilizes metal with k = 20 W/m-K, th = 0.5 mm, a = 20 mm, and b = 15 mm (note that a and b are defined as the half-width and half-height of the heater plate, respectively, and th is the thickness as shown in Figure P2-3). The heater bar temperature is maintained at T_h = 40ºC and the fluid temperature is T_∞ = 20ºC. The nominal value of the average heat transfer coefficient is $\bar{h}$ = 50 W/m^2-K.

a.) Develop an analytical model that can predict the temperature distribution in the plate under these nominal conditions.

b.) The measured quantity is the rate of heat transfer to the plate from the heater ($\dot{q}_h$) and therefore the relationship between $\dot{q}_h$ and $\bar{h}$ (the quantity that is inferred from the heater power) determines how useful the instrument is. Determine the heater power.

c.) If the uncertainty in the measurement of the heater power is $\delta\dot{q}_h$ = 0.01 W, estimate the uncertainty in the measured heat transfer coefficient ($\delta\bar{h}$).

2.2-4 Figure P2.2-4(a) illustrates a proposed device to measure the local heat transfer coefficient from a surface undergoing a boiling heat transfer process. Micro-scale heaters and temperature sensors are embedded in a substrate in a regularly spaced array, as shown.

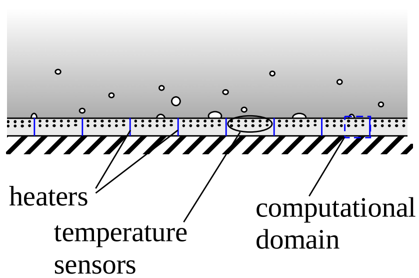

Figure P2.2-4(a): An array of micro-scale heaters and temperature sensors embedded in a substrate.

The heaters are activated, producing a heat flux that is removed primarily from the surface of the substrate exposed to evaporating fluid. The temperature sensors are embedded in sets of two located very near the surface. Each set of thermocouples are used to infer the local heat flux to the surface ($\dot{q}''_s$) and the surface temperature (T_s); these quantities are sufficient to measure the heat transfer coefficient. A half-symmetry model of the region of the substrate between two adjacent heaters (see Figure 2.2-4(a)) is shown in Figure P2.2-4(b).

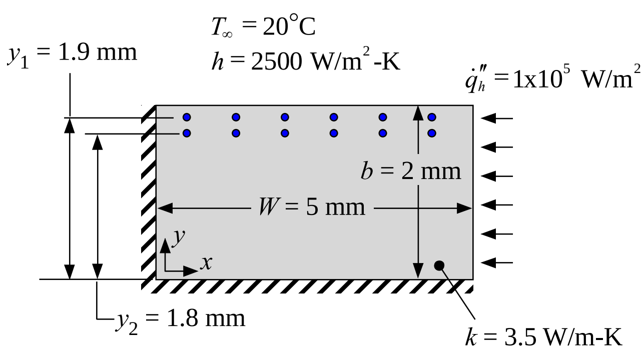

Figure P2.2-4(b): A half-symmetry model of the region of the substrate between two adjacent heaters.

The thickness of the substrate is b = 2 mm and the half-width between adjacent heaters is W = 5 mm. The substrate has conductivity k = 3.5 W/m-K and you may assume that the presence of the temperature sensors does not affect the temperature distribution. The heat flux exposed to the computational domain at $x = W$ is $\dot{q}''_h$ = 1x10^5 W/m^2. The heat transfer coefficient between the evaporating fluid at T_∞ = 20ºC and the surface is h = 2500 W/m^2-K.

a.) Develop a separation of variables solution based on the computational domain shown in Figure P2.2-4(b). Implement your solution in EES.

b.) Prepare a contour plot of the temperature distribution in the substrate.

The position of temperature sensors #1 and #2 at a particular value of x are y_1 = 1.9 mm and y_2 = 1.8 mm, respectively (see Figure P2.2-4(b)). The surface temperature measurement extracted from these measured temperatures is associated with a linear extrapolation to the surface at y = 0:

$$T_{s,m} = T_2 + \left(T_1 - T_2\right)\frac{\left(b - y_2\right)}{\left(y_1 - y_2\right)} \tag{1}$$

The heat flux measurement extracted from these measured temperatures is obtained from Fourier's law according to:

$$\dot{q}''_{s,m} = k\frac{\left(T_2 - T_1\right)}{\left(y_1 - y_2\right)} \tag{2}$$

c.) What is the heat transfer coefficient measured by the device at x/W = 0.5? That is, based on the temperatures T_1 and T_2 predicted by your model at x/W = 0.5, calculate the measured heat transfer coefficient according to:

$$h_m = \frac{\dot{q}''_{s,m}}{\left(T_{s,m} - T_\infty\right)} \tag{3}$$

and determine the discrepancy of your measurement relative to the actual heat transfer coefficient.

d.) Plot the % error associated with the device configuration (i.e., the discrepancy between the measured and actual heat transfer coefficient from part (c)) as a function of axial position, x.

2.2-5 Three heater blocks provide heat to the back-side of a fin array that must be tested, as shown in Figure P2.2-5.

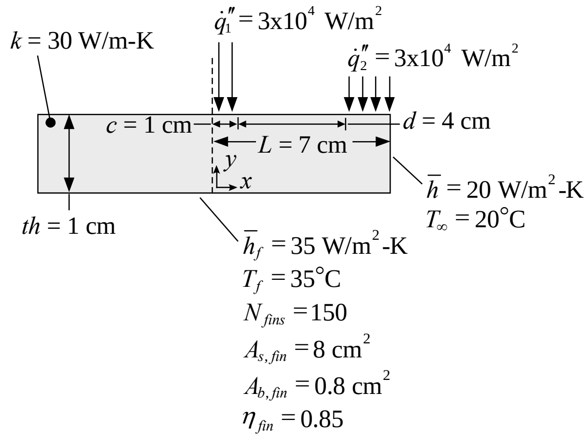

Figure P2.2-5: An array of fins installed on a base plate energized by three heater blocks.

The fins are installed on a base plate that has half-width of L= 7 cm, thickness of th = 1 cm, and width W = 20 cm (into the page). The base plate material has conductivity k = 30 W/m-K. The edge of the base plate (at $x = L$) is exposed to air at T_∞ = 20°C with heat transfer coefficient $\bar{h}$ = 20 W/m^2-K. The middle of the plate (at x = 0) is a line of symmetry and can be modeled as being adiabatic. The bottom of the plate (at y = 0) has an array of N_{fin} = 150 fins installed. Each fin has surface area $A_{s,fin}$= 8 cm^2, base area $A_{b,fin}$ = 0.8 cm^2, and efficiency η_{fin} = 0.85. The fin and the base material are exposed to fluid at T_f = 35°C with heat transfer coefficient $\bar{h}_f$ = 20 W/m^2-K. The top of the plate (at $y = th$) is exposed to the heat flux from the heater blocks. The heat flux is distributed according to:

$$\dot{q}''_{y=th} = \begin{cases} \dot{q}_1'' & \text{if } x < c \\ 0 & \text{if } c < x < (d+c) \\ \dot{q}_2'' & \text{if } x > (d+c) \end{cases}$$

where $\dot{q}_1''$ = 3x10^4 W/m^2, $\dot{q}_2''$ = 3x10^4 W/m^2, c = 1 cm and d = 4 cm.

a.) Determine an effective heat transfer coefficient that can be applied to the surface at $y = th$ in order to capture the combined effect of the fins and the unfinned base area.

b.) Develop a separation of variables solution for the temperature distribution within the fin base material.

c.) Prepare a plot showing the temperature as a function of x at various values of y.

d.) The goal of the base plate is to provide an uniform heat flow to each fin. Assess the performance of the base plate by plotting the rate of heat flux transferred to the fluid as a function of x at y = 0.

e.) Overlay on your plot for (d) the rate of heat flux transferred to the fluid for various values of the base plate conductivity.

2.2-6 (2-4 in text) A laminated composite structure is shown in Figure P2.2-6.

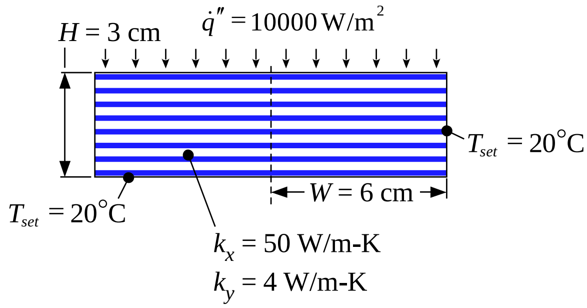

Figure P2.2-6: Composite structure exposed to a heat flux.

The structure is anisotropic. The effective conductivity of the composite in the x-direction is k_x = 50 W/m-K and in the y-direction it is k_y = 4 W/m-K. The top of the structure is exposed to a heat flux of $\dot{q}''$ = 10,000 W/m^2. The other edges are maintained at T_{set} = 20°C. The height of the structure is H = 3 cm and the half-width is W = 6 cm.

a.) Develop a separation of variables solution for the 2-D steady-state temperature distribution in the composite.

b.) Prepare a contour plot of the temperature distribution.

Section 2.3: Advanced Separation of Variables Solutions

2.3-1 (2-5 in text) Figure P2.3-1 illustrates a pipe that connects two tanks of liquid oxygen on a spacecraft. The pipe is subjected to a heat flux, $\dot{q}''$ = 8,000 W/m^2, which can be assumed to be uniformly applied to the outer surface of the pipe and entirely absorbed. Neglect radiation from the surface of the pipe to space. The inner radius of the pipe is r_{in} = 6 cm, the outer radius of the pipe is r_{out} = 10 cm, and the half-length of the pipe is L = 10 cm. The ends of the pipe are attached to the liquid oxygen tanks and therefore are at a uniform temperature of T_{LOx} = 125 K. The pipe is made of a material with a conductivity of k = 10 W/m-K. The pipe is empty and therefore the internal surface can be assumed to be adiabatic.

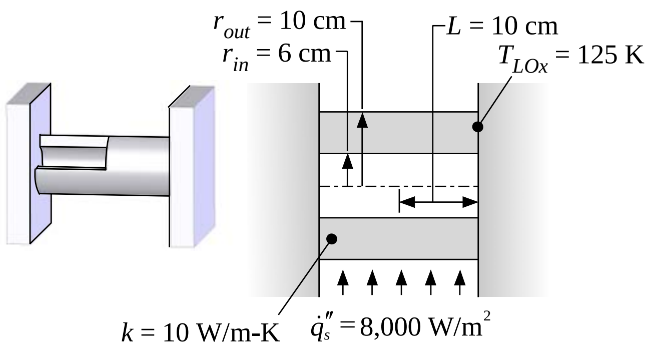

Figure P2.3-1: Cryogen transfer pipe connecting two liquid oxygen tanks.

a.) Develop an analytical model that can predict the temperature distribution within the pipe. Prepare a contour plot of the temperature distribution within the pipe.

2.3-2 (2-6 in text) Figure P2.3-2 illustrates a cylinder that is exposed to a concentrated heat flux at one end.

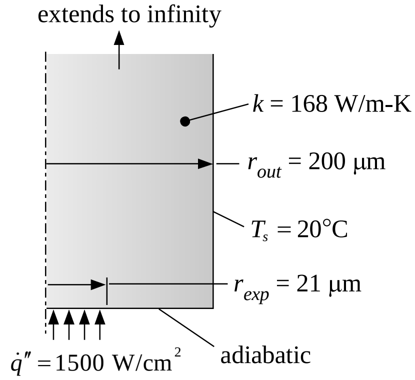

Figure P2.3-2: Cylinder exposed to a concentrated heat flux at one end.

The cylinder extends infinitely in the *x*-direction. The surface at $x = 0$ experiences a uniform heat flux of $\dot{q}'' = 1500\ \text{W/cm}^2$ for $r < r_{exp} = 21\ \mu\text{m}$ and is adiabatic for $r_{exp} < r < r_{out}$ where $r_{out} = 200\ \mu\text{m}$ is the outer radius of the cylinder. The outer surface of the cylinder is maintained at a uniform temperature of $T_s = 20°\text{C}$. The conductivity of the cylinder material is $k = 168$ W/m-K.

a.) Develop a separation of variables solution for the temperature distribution within the cylinder. Plot the temperature as a function of radius for various values of *x*.

b.) Determine the average temperature of the cylinder at the surface exposed to the heat flux.

c.) Define a dimensionless thermal resistance between the surface exposed to the heat flux and T_s. Plot the dimensionless thermal resistance as a function of r_{out}/r_{in}.

d.) Show that your plot from (c) does not change if the problem parameters (e.g., T_s, k, etc.) are changed.

2.3-3 A disk-shaped window in an experiment is shown in Figure P2.3-3.

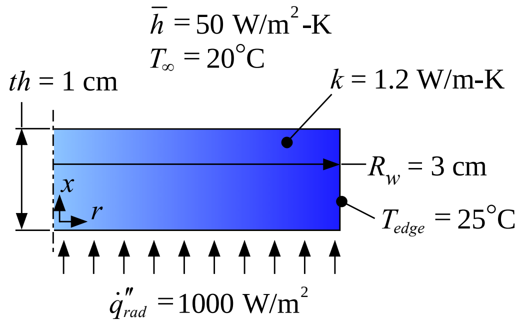

Figure P2.3-3: Window.

The inside of the window (the surface at x = 0) is exposed to vacuum and therefore does not experience any convection. However, this surface is exposed to a radiation heat flux $\dot{q}''_{rad}$ = 1000 W/m^2. The window is assumed to be completely opaque to this radiation and therefore it is absorbed at x = 0. The edge of the window at R_w = 3 cm is maintained at a constant temperature T_{edge} = 25°C. The outside of the window (the surface at x = th) is cooled by air at T_∞ = 20°C with heat transfer coefficient $\bar{h}$ = 50 W/m^2-K. The conductivity of the window material is k = 1.2 W/m-K.

a.) Is the extended surface approximation appropriate for this problem? That is, can the temperature in the window be approximated as being 1-D in the radial direction? Justify your answer.

b.) Assume that your answer to (a) is no. Develop a 2-D separation of variables solution to this problem.

c.) Plot the temperature as a function of r for various values of x.

d.) Prepare a contour plot of the temperature in the window.

2.3-4 Reconsider Problem 2.3-3. The window is not opaque to the radiation but does absorb some of it. The radiation that is absorbed is transformed to thermal energy. The volumetric rate of thermal energy generation is given by: $\dot{g}''' = \dot{q}''_{rad}\,\alpha \exp\left(-\alpha\, x\right)$ where α = 100 m^{-1} is the absorption coefficient. The radiation that is not absorbed is transmitted. Otherwise the problem remains the same.

a.) Develop a separation of variables solution to this problem using the techniques discussed in Section 2.3.

b.) Plot the temperature as a function of *r* for various values of *x*.

c.) Show that your solution limits to the solution from Problem 2.3-3 in the limit that $\alpha \to \infty$.

Section 2.4: Superposition

2.4-1 (2-7 in text) The plate shown in Figure P2.4-1 is exposed to a uniform heat flux $\dot{q}'' = 1\text{x}10^5$ W/m^2 along its top surface and is adiabatic at its bottom surface. The left side of the plate is kept at T_L = 300 K and the right side is at T_R = 500 K. The height and width of the plate are H = 1 cm and W = 5 cm, respectively. The conductivity of the plate is k = 10 W/m-K.

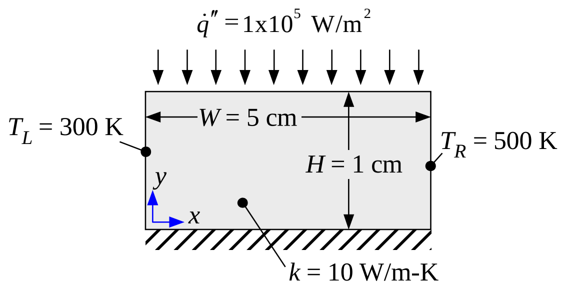

Figure P2.4-1: Plate.

a.) Derive an analytical solution for the temperature distribution in the plate.

b.) Implement your solution in EES and prepare a contour plot of the temperature.

Section 2.5: Numerical Solutions to Steady-State 2-D Problems using EES

2.5-1 (2-8 in text) Figure P2.5-1 illustrates an electrical heating element that is affixed to the wall of a chemical reactor. The element is rectangular in cross-section and very long (into the page). The temperature distribution within the element is therefore two-dimensional, $T(x, y)$. The width of the element is a = 5.0 cm and the height is b = 10.0 cm. The three edges of the element that are exposed to the chemical (at x = 0, y = 0, and x = a) are maintained at a temperature T_c = 200°C while the upper edge (at y = b) is affixed to the well-insulated wall of the reactor and can therefore be considered adiabatic. The element experiences a uniform volumetric rate of thermal energy generation, $\dot{g}'''$ = $1\text{x}10^6$ W/m^3. The conductivity of the material is k = 0.8 W/m-K.

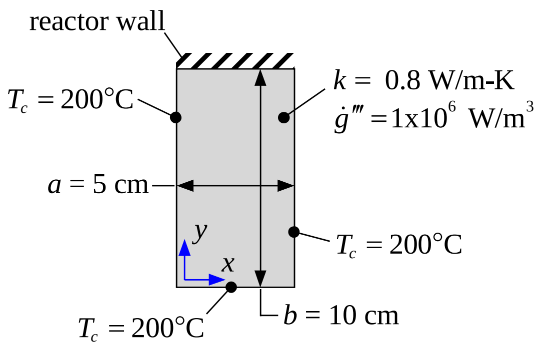

Figure P2.5-1: Electrical heating element.

a.) Develop a 2-D numerical model of the element using EES.

b.) Plot the temperature as a function of x at various values of y. What is the maximum temperature within the element and where is it located?

c.) Prepare a reality check to show that your solution behaves according to your physical intuition. That is, change some aspect of your program and show that the results behave as you would expect (clearly describe the change that you made and show the result).

2.5-2 Figure P2.5-2 illustrates a composite material that is being machined on a lathe.

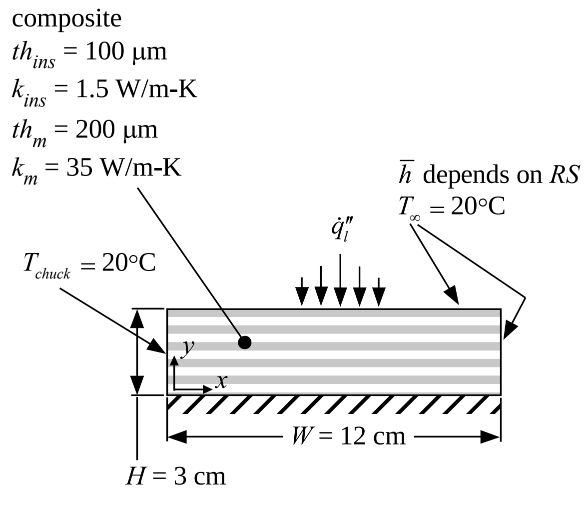

Figure P2.5-2: Composite material being machined on a lathe.

The composite is composed of alternating layers of insulating material and metal. The insulating layers have thickness th_{ins}= 100 μm and conductivity k_{ins} = 1.5 W/m-K. The metal layers have thickness th_m = 200 μm and conductivity k_m = 35 W/m-K. The workpiece is actually cylindrical and rotating. However, because the radius is large relative to it thickness and there are no circumferential variations we can model the workpiece as a 2-D problem in Cartesian coordinates, x and y, as shown in Figure 2.5-2. The width of the workpiece is W = 12 cm and the thickness is H = 3 cm. The left surface of the workpiece at x = 0 is attached to the chuck and therefore maintained at T_{chuck} = 20°C. The inner surface at y = 0 is insulated. The outer surface (at y = H) and right surface (at x = W) are exposed to air at T_{∞} = 20°C with heat transfer coefficient $\overline{h}$ that depends on the rotational speed of the chuck, RS in rev/min, according to:

$$\overline{h} = 2\left[\frac{\text{W min}^2}{\text{m}^2\,\text{K rev}^2}\right] RS^2$$

In order to extend the life of the tool used for the machining process, the workpiece is preheated by applying laser power to the outer surface. The heat flux applied by the laser depends on the rotational speed and position according to:

$$\dot{q}_l'' = a\,RS\exp\left[-\left(\frac{x - x_c}{pw}\right)^2\right]$$

where a = 5000 W-min/m^2-rev, x_c = 8 cm, and pw = 1 cm.

a.) What is the effective thermal conductivity of the composite in the x- and y-directions?

b.) Develop a 2-D numerical model of the workpiece in EES. Plot the temperature as a function of x at various values of y, including at least y = 0, H/2, and H.

c.) Plot the maximum temperature in the workpiece as a function of the rotational speed, *RS*. If the objective is to preheat the material to its maximum possible temperature, then what is the optimal rotational speed?

2.5-3 Figure P2.5-3 illustrates a heater that extends from a wall into fluid that is to be heated.

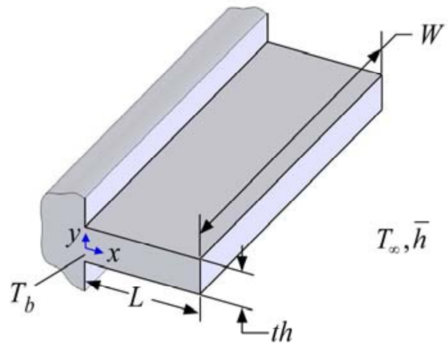

Figure P2.5-3: Heater.

Assume that the tip of the heater is insulated and that the width (*W*) is much larger than the thickness (*th*) so that convection from the edges can be neglected. The length of the heater is L = 5.0 cm. The base temperature is T_b = 20ºC and the heater experiences convection with fluid at T_∞= 100ºC with average heat transfer coefficient, $\overline{h}$ = 100 W/m^2-K. The heater is th = 3.0 cm thick and has conductivity k = 1.5 W/m-K. The heater experiences a uniform volumetric generation of $\dot{g}'''$ = 5x10^5 W/m^3.

a.) Develop a numerical solution for the temperature distribution in the heater using a finite difference technique.

b.) Use the numerical solution to predict and plot the temperature distribution in the heater.

c.) Use the numerical solution to predict the heater efficiency; the heater efficiency is defined as the ratio of the rate of heat transfer to the fluid to the total rate of thermal energy generation in the fin.

d.) Plot the heater efficiency as a function of the length for various values of the thickness. Explain your plot.

Section 2.6: Finite-Difference Solutions to Steady-State 2-D Problems using MATLAB

2.6-1 (2-9 in text) Figure P2.6-1 illustrates a cut-away view of two plates that are being welded together. Both edges of the plate are clamped and effectively held at temperatures T_s = 25°C. The top of the plate is exposed to a heat flux that varies with position x, measured from joint, according to: $\dot{q}''_m(x) = \dot{q}''_j \exp(-x/L_j)$ where $\dot{q}''_j$ =1x10^6 W/m^2 is the maximum heat flux (at the joint, x = 0) and L_j = 2.0 cm is a measure of the extent of the heat flux. The back side of the plates are exposed to liquid cooling by a jet of fluid at T_f = -35°C with $\bar{h}$ = 5000 W/m^2-K. A half-symmetry model of the problem is shown in Figure P2.6-1. The thickness of the plate is b = 3.5 cm and the width of a single plate is W = 8.5 cm. You may assume that the welding process is steady-state and 2-D. You may neglect convection from the top of the plate. The conductivity of the plate material is k = 38 W/m-K.

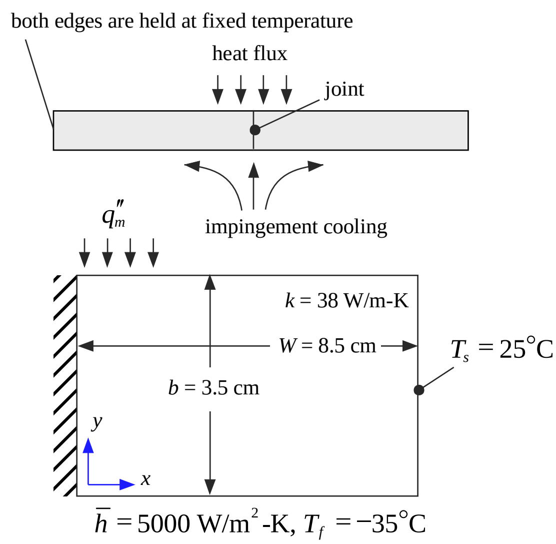

Figure P2.6-1: Welding process and half-symmetry model of the welding process.

a.) Develop a separation of variables solution to the problem (note, this was done previously in Problem 2.2-1). Implement the solution in EES and prepare a plot of the temperature as a function of x at y = 0, 1.0, 2.0, 3.0, and 3.5 cm.

b.) Prepare a contour plot of the temperature distribution.

c.) Develop a numerical model of the problem. Implement the solution in MATLAB and prepare a contour or surface plot of the temperature in the plate.

d.) Plot the temperature as a function of x at y = 0, b/2, and b and overlay on this plot the separation of variables solution obtained in part (a) evaluated at the same locations.

2.6-2 Prepare a solution to Problem 2.3-3 using a finite difference technique.

a.) Plot the temperature as a function of *r* for various values of *x*.

b.) Prepare a contour plot of the temperature in the window.

c.) Verify that your solution agrees with the analytical solution from Problem 2.3-3.

2.6-3 Prepare a solution to Problem 2.3-4 using a finite difference technique.

a.) Plot the temperature as a function of *r* for various values of *x*.

b.) Prepare a contour plot of the temperature in the window.

c.) Verify that your solution agrees with the analytical solution from Problem 2.3-4.

Section 2.7: Finite-Element Solutions to Steady-State 2-D Problems using FEHT

2.7-1 (2-10 in text) Figure P2.7-1(a) illustrates a double paned window. The window consists of two panes of glass each of which is tg = 0.95 cm thick and W = 4 ft wide by H = 5 ft high. The glass panes are separated by an air gap of g = 1.9 cm. You may assume that the air is stagnant with k_a = 0.025 W/m-K. The glass has conductivity k_g = 1.4 W/m-K. The heat transfer coefficient between the inner surface of the inner pane and the indoor air is $\bar{h}_{in}$ = 10 W/m^2-K and the heat transfer coefficient between the outer surface of the outer pane and the outdoor air is $\bar{h}_{out}$ = 25 W/m^2-K. You keep your house heated to T_{in} = 70°F.

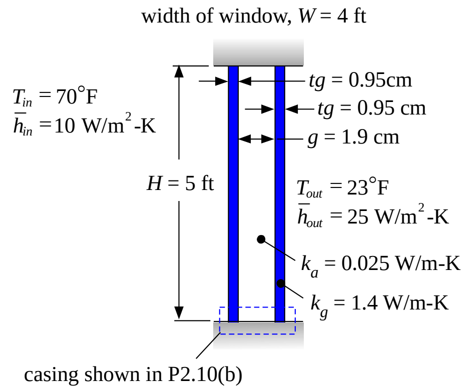

Figure P2.7-1(a): Double paned window.

The average heating season in Madison lasts about $time$ = 130 days and the average outdoor temperature during this time is T_{out} = 23°F. You heat with natural gas and pay, on average, ec = 1.415 \$/therm (a therm is an energy unit =1.055x10^8 J).

a.) Calculate the average rate of heat transfer through the double paned window during the heating season.

b.) How much does the energy lost through the window cost during a single heating season?

There is a metal casing that holds the panes of glass and connects them to the surrounding wall, as shown in Figure P2.7-1(b). Because the metal casing is high conductivity, it seems likely that you could lose a substantial amount of heat by conduction through the casing (potentially negating the advantage of using a double paned window). The geometry of the casing is shown in Figure P2.7-1(b); note that the casing is symmetric about the center of the window.

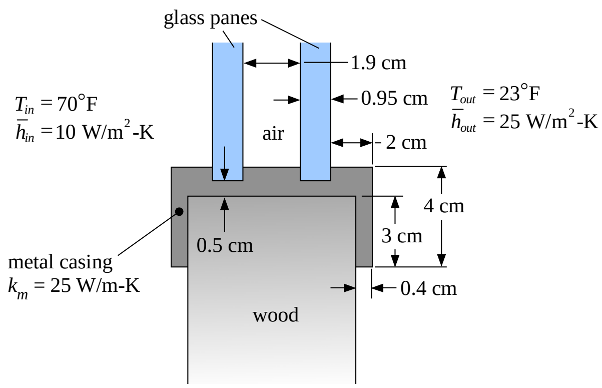

Figure P2-10(b) Metal casing.

All surfaces of the casing that are adjacent to glass, wood, or the air between the glass panes can be assumed to be adiabatic. The other surfaces are exposed to either the indoor or outdoor air.

c.) Prepare a 2-D thermal analysis of the casing using FEHT. Turn in a print out of your geometry as well as a contour plot of the temperature distribution. What is the rate of energy lost via conduction through the casing per unit length (W/m)?

d.) Show that your numerical model has converged by recording the rate of heat transfer per length for several values of the number of nodes.

e.) How much does the casing add to the cost of heating your house?

2.7-2 Relatively hot gas flows out of the stack of a power plant. The energy associated with these combustion products is useful for providing hot water or other low grade energy. The system shown in Figure P2.7-2(a) has been developed to recover some of this energy.

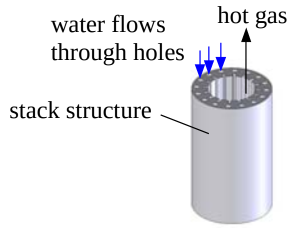

Figure P2.7-2(a): Energy recovery system for hot exhaust gas.

The system is fabricated from a high temperature material in the form of a ring; 16 fluid channels are integrated with the stack in a circular array. The water to be heated flows through these channels. The inner surface of the liner is finned to increase its surface area and is exposed to the hot gas while the outer surface is cooled externally by ambient air. A cut-away view of the stack is shown in Figure P2.7-2(b).

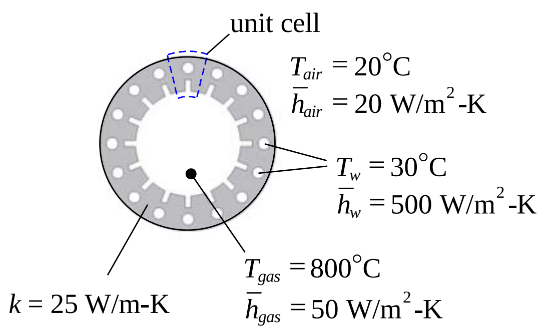

Figure P2.7-2(b): Problem specification for the energy recovery problem.

At a particular section, the water can be modeled as being at a uniform temperature of T_w = 30°C with a heat transfer coefficient, $\overline{h}_w$ = 500 W/m^2-K. The hot gas is at T_{gas} = 800°C with a heat transfer coefficient, $\overline{h}_{gas}$ = 50 W/m^2-K. The ambient air external to the stack is at T_{air} = 20°C and $\overline{h}_{air}$ = 20 W/m^2-K. The stack material has conductivity k = 25 W/m-K.

The liner geometry is relatively complex and includes curved segments as well as straight sections. Only a single unit cell of the structure (see Figure P2.7-2(b)) needs to be simulated. FEHT does not allow curved sections to be simulated; rather, a curved section must be approximated as a polygon. Rather than attempting to draw the geometry manually, it is preferable to import a drawing (e.g., from a computer aided drawing package) and trace the drawing in FEHT. More advanced finite element tools will have automated processes for importing geometry from various sources. A drawing of the unit cell with a scale can be copied onto the clipboard (from the website for this text, using Microsoft Powerpoint) and pasted into FEHT.

a.) Use FEHT to develop a finite element model of the stack and determine the heat transfer to the water per unit length of stack.
b.) Verify that your solution has converged numerically.
c.) Sanity check your results against a simple model.

2.7-3 (2-11 in text) A radiator panel extends from a spacecraft; both surfaces of the radiator are exposed to space (for the purposes of this problem it is acceptable to assume that space is at 0 K); the emittance of the surface is $\varepsilon = 1.0$. The plate is made of aluminum (k = 200 W/m-K and ρ = 2700 kg/m^3) and has a fluid line attached to it, as shown in Figure 2.7-3(a). The half-width of the plate is a=0.5 m wide while the height of the plate is b=0.75m. The thickness of the plate is a design variable and will be varied in this analysis; begin by assuming that the thickness is th = 1.0 cm. The fluid lines carry coolant at T_c = 320 K. Assume that the fluid temperature is constant although the fluid temperature will actually decrease as it transfers heat to the radiator. The combination of convection and conduction through the panel-to-fluid line mounting leads to an effective heat transfer coefficient of h = 1,000 W/m^2-K over the 3.0 cm strip occupied by the fluid line.

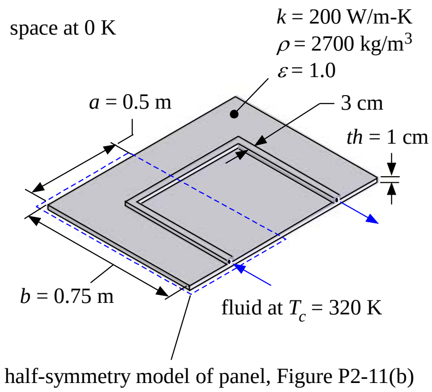

Figure 2.7-3(a): Radiator panel

The radiator panel is symmetric about its half-width and the critical dimensions that are required to develop a half-symmetry model of the radiator are shown in Figure 2.7-3(b). There are three regions associated with the problem that must be defined separately so that the surface conditions can be set differently. Regions 1 and 3 are exposed to space on both sides while Region 2 is exposed to the coolant fluid one side and space on the other; for the purposes of this problem, the effect of radiation to space on the back side of Region 2 is neglected.

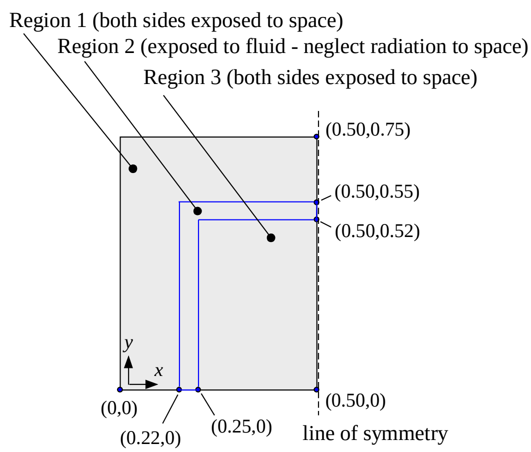

Figure 2.7-3(b): Half-symmetry model.

a.) Prepare a FEHT model that can predict the temperature distribution over the radiator panel.
b.) Export the solution to EES and calculate the total heat transferred from the radiator and the radiator efficiency (defined as the ratio of the radiator heat transfer to the heat transfer from the radiator if it were isothermal and at the coolant temperature).
c.) Explore the effect of thickness on the radiator efficiency and mass.

2.7-4 Gas turbine power cycles are used for the generation of power; the size of these systems can range from 10's of kWs for the microturbines that are being installed on-site at some commercial and industrial locations to 100's of MWs for natural gas fired power plants. The efficiency of a gas turbine power plant increases with the temperature of the gas entering from the combustion chamber; this temperature is constrained by the material limitations of the turbine blades which tend to creep (i.e., slowly grow over time) in the high temperature environment in their high centrifugal stress state. One technique for achieving high gas temperatures is to cool the blades internally; often the air is bled through the blade surface using a technique called transpiration. A simplified version of a turbine blade that will be analyzed in this problem is shown in Figure P2.7-4.

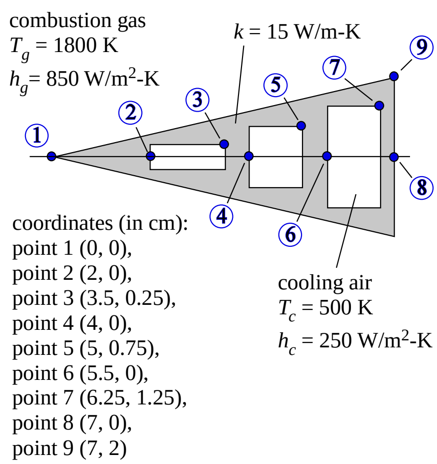

Figure 2.7-4: A simplified schematic of an air cooled blade.

The high temperature combustion gas is at T_g = 1800 K and the heat transfer coefficient between the gas and blade external surface is h_g = 850 W/m^2-K. The blades are cooled by three internal air passages. The cooling air in the passages is at T_c = 500 K and the air-to-blade heat transfer coefficient is h_a= 250 W/m^2-K. The blade material has conductivity k = 15 W/m-K. The coordinates of the points required to define the geometry are indicated in Figure P2.7-4.

a.) Generate a ½ symmetry model of the blade in FEHT. Generate a figure showing the temperature distribution in the blade predicted using a very crude mesh.

b.) Refine your mesh and keep track of the temperature experienced at the trailing edge of the blade (i.e., at position 9 in Figure P2.7-4) as a function of the number of nodes in your mesh. Prepare a plot of this data that can be used to establish that your model has converged to the correct solution.

c.) Do your results make sense? Use a very simple, order-of-magnitude analysis based on thermal resistances to decide whether your predicted blade surface temperature is reasonable (hint – there are three thermal resistances that govern the behavior of the blade, estimate each one and show that your results are approximately correct given these thermal resistances).

2.7-5 a.) Show how the construction of the finite element problem changes with the addition of volumetric generation.

b.) Re-solve the problem discussed in Section 2.7.2 assuming that the material experiences a volumetric generation rate of $\dot{g}''' = 1\text{x}10^3$ W/m^3, as shown in Figure P2.7-5.

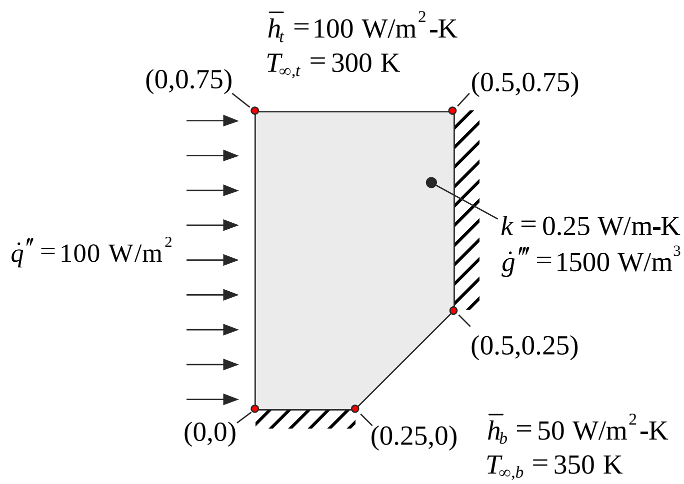

Figure P2.7-5: Two-dimensional conduction problem used to illustrate the finite element solution with generation. The coordinates of points are shown in m.

2.7-6 Figure P2.7-6 illustrates a tube in a water-to-air heat exchanger with a layer of polymer coating that can be easily etched away in order to form an array of fin-like structures that increase the surface area exposed to air.

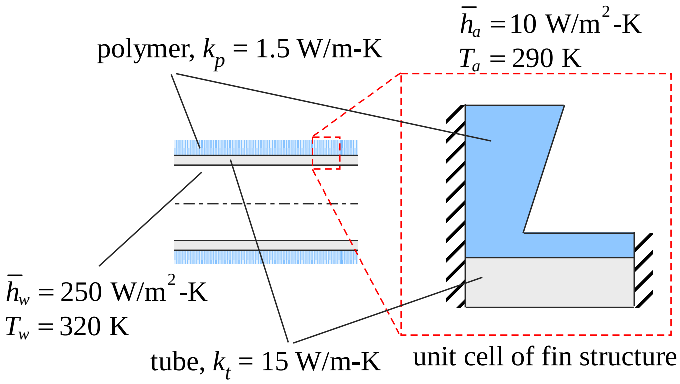

Figure P2.7-6: Tube coated with polymer and a unit cell showing fin-like structures etched into polymer.

The water flowing through the tube has temperature T_w = 320 K and heat transfer coefficient $\bar{h}_w$ = 250 W/m^2-K. The air has temperature T_a = 290 K and heat transfer coefficient $\bar{h}_a$ = 10 W/m^2-K. The thermal conductivity of the polymer and tube material is k_p = 1.5 W/m-K and k_t = 15 W/m-K, respectively.

a.) Generate a finite element solution for the temperature distribution within the unit cell shown in Figure 2.7-6 using the mesh shown in Figure 2.7-6(b). The coordinates of the nodes are listed in Table P2.7-6.

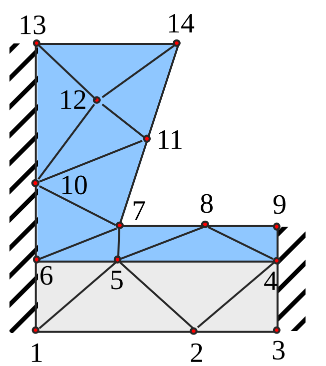

Figure P2.7-6(b): Mesh for finite element solution.

Table 2.7-6: Coordinates of nodes in Figure 2.7-6(b).

Node	x-coord. (m)	y-coord. (m)	Node	x-coord. (m)	y-coord. (m)
1	0	0	8	0.01	0.01
2	0.01	0	9	0.015	0.01
3	0.015	0	10	0	0.015
4	0.015	0.008	11	0.007	0.02
5	0.005	0.008	12	0.005	0.025
6	0	0.008	13	0	0.03
7	0.006	0.01	14	0.009	0.03

b.) Plot the average heat flux at the internal surface of the tube as a function of the air-side heat transfer coefficient with and without the polymer coating. You should see a cross-over point where it becomes disadvantageous to use the polymer coating; explain this.

2.7-7 Figure P2.7-7 illustrates a power electronics chip that is used to control the current to a winding of a motor.

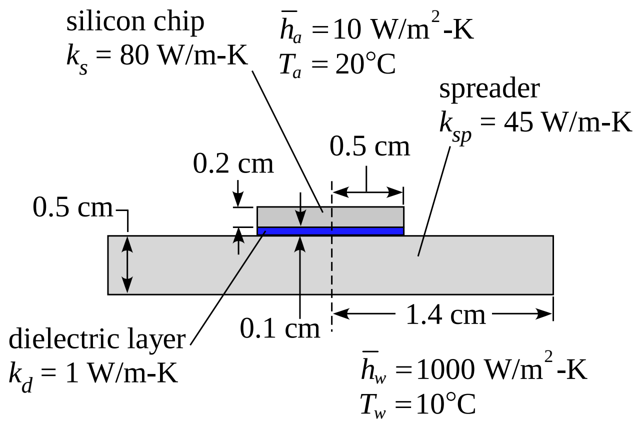

Figure P2.7-7: Power electronics chip.

The silicon chip has dimensions 0.2 cm by 0.5 cm and conductivity k_s = 80 W/m-K. A generation of thermal energy occurs due to losses in the chip; the thermal energy generation can be modeled as being uniformly distributed with a value of $\dot{g}'''$ = 1x10^8 W/m^3 in the upper 50% of the silicon. The chip is thermally isolated from the spreader by a dielectric layer with thickness 0.1 cm and conductivity k_d = 1 W/m-K. The spreader has dimension 0.5 cm by 1.4 cm and conductivity k_{sp} = 45 W/m-K. The external surfaces are all air cooled with $\bar{h}_a$ = 10 W/m^2-K and T_a = 20ºC except for the bottom surface of the spreader which is water cooled with $\bar{h}_w$ = 1000 W/m^2-K and T_w = 10ºC.

a.) Develop a numerical model of the system using FEHT.
b.) Plot the maximum temperature in the system as a function of the number of nodes.
c.) Develop a simple sanity check of your results using a resistance network.

2.7-8 Figure P2.7-8(a) illustrates a heat exchanger in which hot fluid and cold fluid flows through alternating rows of square channels that are installed in a piece of material.

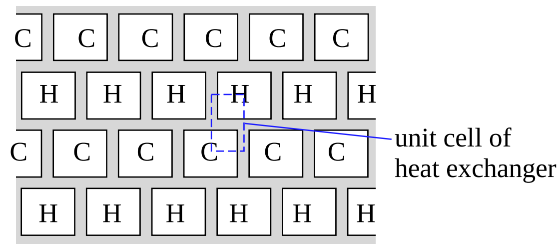

Figure P2.7-8(a): Heat exchanger.

You are analyzing this heat exchanger and will develop a model of the unit cell shown in Figure P2.7-8(a) and illustrated in more detail in Figure 2.7-8(b).

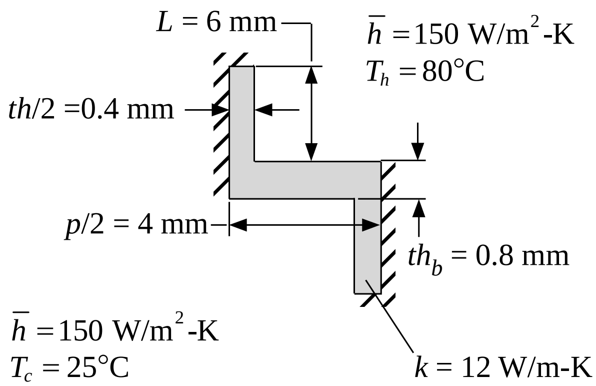

Figure 2.7-8(b): Details of unit cell shown in Figure 2.7-8(a).

The metal struts separating the square channels form fins. The length of the fin (the half-width of the channel) is L = 6 mm and the fin thickness is th = 0.8 mm. The thickness of the material separating the channels is th_b = 0.8 mm. The distance between adjacent fins is p = 8 mm. The channel structure for both sides (hot and cold) are identical. The conductivity of the metal is k = 12 W/m-K. The hot fluid has temperature T_h = 80ºC and heat transfer coefficient $\bar{h}$ = 150 W/m^2-K. The cold fluid has temperature T_c = 25ºC and the same heat transfer coefficient.

a.) Prepare a numerical model of the unit cell shown in Figure 2.7-8(b) using FEHT.

b.) Plot the rate of heat transfer from the hot fluid to the cold fluid within the unit cell as a function of the number of nodes.

c.) Develop a simple model of the unit cell using a resistance network and show that your result from (b) makes sense.

Section 2.8: Resistance Approximations for Conduction Problems

2.8-1 (2-12 in text) There are several cryogenic systems that require a "thermal switch", a device that can be used to control the thermal resistance between two objects. One class of thermal switch is activated mechanically and an attractive method of providing mechanical actuation at cryogenic temperatures is with a piezoelectric stack; unfortunately, the displacement provided by a piezoelectric stack is very small, typically on the order of 10 microns. A company has proposed an innovative design for a thermal switch, shown in Figure P2.8-1(a). Two blocks are composed of th = 10 μm laminations that are alternately copper (k_{Cu} = 400 W/m-K) and plastic (k_p = 0.5 W/m-K). The thickness of each block is L = 2.0 cm in the direction of the heat flow. One edge of each block is carefully polished and these edges are pressed together; the contact resistance associated with this joint is $R''_c = 5\text{x}10^{-4}$ K-m^2/W.

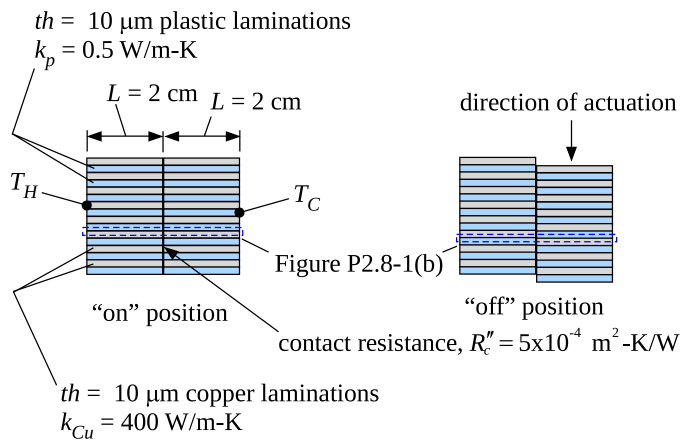

Figure P2.8-1(a): Thermal switch in the "on" and "off" positions.

Figure P2.8-1(a) shows the orientation of the two blocks when the switch is in the "on" position; notice that the copper laminations are aligned with one another in this configuration which provides a continuous path for heat through high conductivity copper (with the exception of the contact resistance at the interface). The vertical location of the right-hand block is shifted by 10 μm to turn the switch "off". In the "off" position, the copper laminations are aligned with the plastic laminations; therefore, the heat transfer is inhibited by low conductivity plastic. Figure P2.8-1(b) illustrates a closer view of half (in the vertical direction) of two adjacent laminations in the "on" and "off" configurations. Note that the repeating nature of the geometry means that it is sufficient to analyze a single lamination set and assume that the upper and lower boundaries are adiabatic.

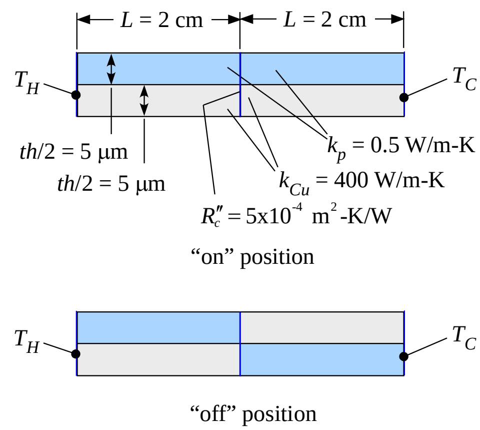

Figure P2.8-1(b): A single set consisting of half of two adjacent laminations in the "on" and "off" positions.

The key parameter that characterizes a thermal switch is the resistance ratio (*RR*) which is defined as the ratio of the resistance of the switch in the "off" position to its resistance in the "on" position. The company claims that they can achieve a resistance ratio of more than 100 for this switch.

a) Estimate upper and lower bounds for the resistance ratio for the proposed thermal switch using 1-D conduction network approximations. Be sure to draw and clearly label the resistance networks that are used to provide the estimates. Use your results to assess the company's claim of a resistance ratio of 100.

b) Provide one or more suggestions for design changes that would improve the performance of the switch (i.e., increase the resistance ratio). Justify your suggestions.

c.) Sketch the temperature distribution through the two parallel paths associated with the adiabatic limit of the switch's operation in the "off" position. Do not worry about the quantitative details of the sketch, just make sure that the qualitative features are correct.

d.) Sketch the temperature distribution through the two parallel paths associated with the adiabatic limit in the "on" position. Again, do not worry about the quantitative details of your sketch, just make sure that the qualitative features are correct.

P2.8-2 (2-13 in text) Figure P2.8-2 illustrates a thermal bus bar that has width W = 2 cm (into the page).

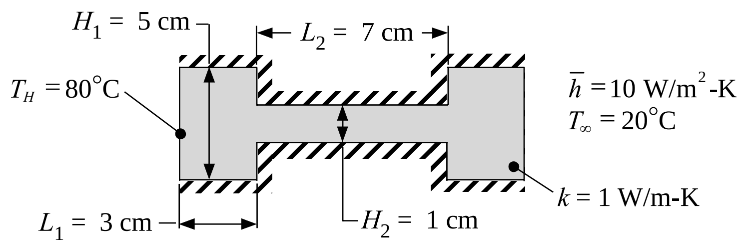

Figure P2.8-2: Thermal bus bar.

The bus bar is made of a material with conductivity k = 1 W/m-K. The middle section is L_2 = 7 cm long with thickness H_2 = 1 cm. The two ends are each L_1 = 3 cm long with thickness H_1 = 3 cm. One end of the bar is held at T_H = 80ºC and the other is exposed to air at T_∞ = 20ºC with $\bar{h}$ = 10 W/m^2-K.

a.) Use FEHT to predict the rate of heat transfer through the bus bar.

b.) Obtain upper and lower bounds for the rate of heat transfer through the bus bar using appropriately defined resistance approximations.

2.8-3 Figure P2.8-3 illustrates a design for a superconducting heat switch.

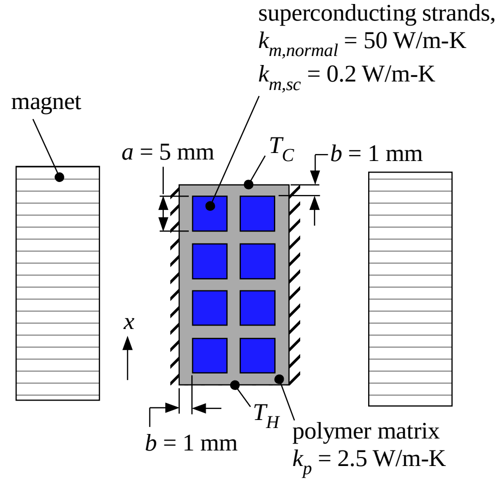

Figure P2.8-3: Superconducting heat switch.

The heat switch is made by embedding eight square superconducting strands in a polymer matrix. The width of switch is W = 10 mm (into the page). The size of the strands are a = 5 mm and the width of polymer that surrounds each strand is b = 1 mm. The conductivity of the polymer is k_p = 2.5 W/m-K. The heat switch is surrounded by a magnet. When the heat switch is on (i.e., the thermal resistance through the switch in the x-direction is low, allowing heat flow from T_H to T_C), the magnet is on. Therefore, the magnetic field tends to drive the superconductors to their normal state where they have a high thermal conductivity, $k_{m,normal}$ = 50 W/m-K. To turn the heat switch off (i.e., to make the thermal resistance through the switch high, preventing heat transfer from T_H to T_C), the magnet is deactivated. The superconductors return to their superconducting state, where they have a low thermal conductivity, $k_{m,sc}$ = 0.2 W/m-K. The edges of the switch are insulated.

a.) Develop a model using a 1-D resistance network that provides a lower bound on the resistance of the switch when it is in its off state (i.e., $k_m = k_{m,sc}$).

b.) Develop a model using a 1-D resistance network that provides an upper bound on the resistance of the switch when it is in its off state (i.e., $k_m = k_{m,sc}$).

c.) Plot your answers from parts (a) and (b) as a function of k_m for $k_{m,sc} < k_m < k_{m,normal}$.

d.) The performance of a heat switch is provided by the resistance ratio; the ratio of the resistance of the switch in its off state to its resistance in the on state. Use your model to provide an upper and lower bound on the resistance ratio of the switch.

e.) Plot the ratio of your answer from part (b) to your answer from part (a) as a function of k_m for $k_{m,sc} < k_m < k_{m,normal}$. Explain the shape of your plot.

Section 2.9: Conduction through Composite Materials

2.9-1 A composite material is formed from laminations of high conductivity material (k_{high} = 100 W/m-K) and low conductivity material (k_{low} = 1 W/m-K) as shown in Figure P2.9-1. Both laminations have the same thickness, *th*.

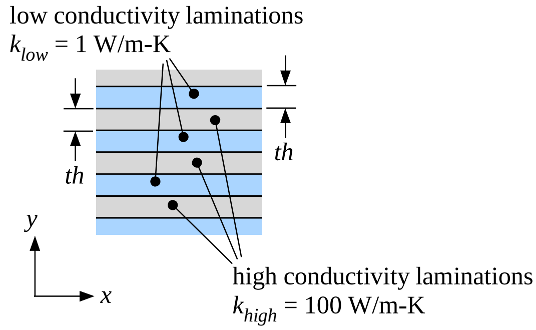

Figure P2.9-1: Composite material formed from high and low conductivity laminations.

a.) Do you expect the equivalent conductivity of the composite to be higher in the *x* or *y* directions? Note from Figure P2.9-1 that the *x*-direction is parallel to the laminations while the *y* direction is perpendicular to the laminations.

b.) Estimate the equivalent conductivity of the composite in the *x*-direction. You should not need to any calculations to come up with a good estimate for this quantity.

2.9-2 (2-14 in text) A laminated stator is shown in Figure P2.9-2. The stator is composed of laminations with conductivity k_{lam} = 10 W/m-K that are coated with a very thin layer of epoxy with conductivity k_{epoxy} = 2.0 W/m-K in order to prevent eddy current losses. The laminations are th_{lam} = 0.5 mm thick and the epoxy coating is 0.1 mm thick (the total amount of epoxy separating each lamination is th_{epoxy} = 0.2 mm). The inner radius of the laminations is r_{in}= 8.0 mm and the outer radius of the laminations is $r_{o,lam}$ = 20 mm. The laminations are surrounded by a cylinder of plastic with conductivity k_p = 1.5 W/m-K that has an outer radius of $r_{o,p}$ = 25 mm. The motor casing surrounds the plastic. The motor casing has an outer radius of $r_{o,c}$ = 35 mm and is composed of aluminum with conductivity k_c = 200 W/m-K.

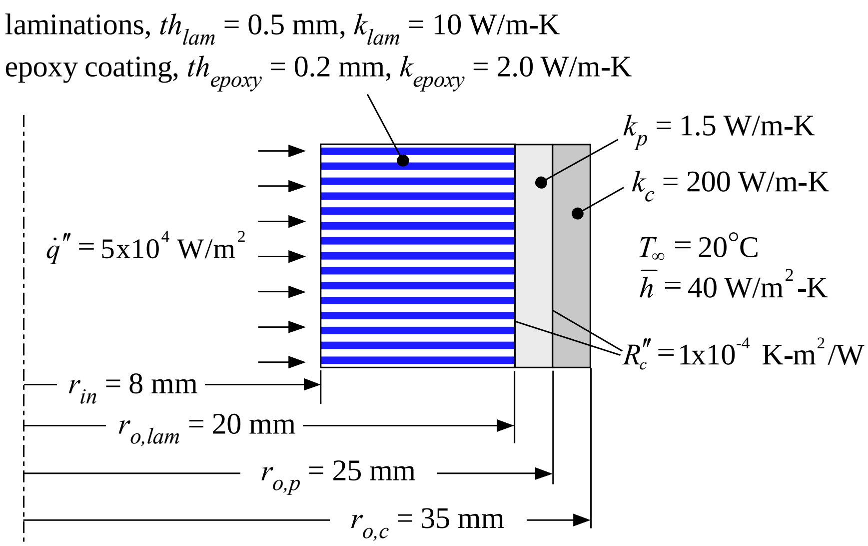

Figure P2.9-2: Laminated stator.

A heat flux associated with the windage loss associated with the drag on the shaft is $\dot{q}''$ = 5x10^4 W/m^2 is imposed on the internal surface of the laminations. The outer surface of the motor is exposed to air at T_∞ = 20°C with a heat transfer coefficient $\overline{h}$ = 40 W/m^2-K. There is a contact resistance R_c'' = 1x10^{-4} K-m^2/W between the outer surface of the laminations and the inner surface of the plastic and the outer surface of the plastic and the inner surface of the motor housing.

a.) Determine an upper and lower bound for the temperature at the inner surface of the laminations (T_{in}).

b.) You need to reduce the internal surface temperature of the laminations and there are a few design options available, including: (1) increase the lamination thickness (up to 0.7 mm), (2) reduce the epoxy thickness (down to 0.05 mm), (3) increase the epoxy conductivity (up to 2.5 W/m-K), or (4) increase the heat transfer coefficient (up to 100 W/m-K). Which of these options do you suggest and why?

2.9-3 Your company manufactures a product that consists of many small metal bars that run through a polymer matrix, as shown in Figure P2.9-3. The material can be used as a thermal path, allowing heat to transfer efficiently in the *z*-direction (the direction that the metal bars run) because the heat can travel without interruption through the metal bars. However, the material blocks heat flow in the *x*- and *y*-directions because the energy must be conducted through the low conductivity polymer. Because the scale of the metal bars is small relative to the size of the composite structure, it is appropriate to model the material as a composite with an effective conductivity that depends on direction.

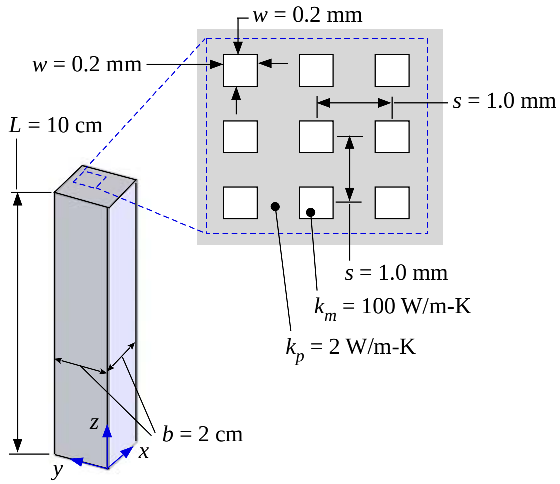

Figure P2.9-3: Composite material.

The metal bars are square with edge width w = 0.2 mm and are aligned with the *z*-direction. The bars are arrayed in a regularly spaced matrix with a center-to-center distance of s = 1.0 mm. The conductivity of the metal is k_m = 100 W/m-K. The length of the material in the *z* direction is L = 10 cm. The polymer fills the space between the bars and has a thermal conductivity k_p = 2.0 W/m-K. The cross-section of the material in the *x*-*y* plane is square with edge width b = 2.0 cm.

a.) Determine the effective conductivity in the *x*- , *y*- , and *z*-directions.

b.) The outer edges of the material are insulated and the faces of the material at z = 0 and $z = L$ are exposed to a convective boundary condition with $\overline{h}$ = 10 W/m^2-K. Is it appropriate to treat this problem as a lumped capacitance problem?

Chapter 2: Two-Dimensional, Steady-State Conduction

Section 2.1: Shape Factors

2.1-1 (2-1 in text) Figure P2.1-1 illustrates two tubes that are buried in the ground behind your house that transfer water to and from a wood burner. The left hand tube carries hot water from the burner back to your house at $T_{w,h}$ = 135°F while the right hand tube carries cold water from your house back to the burner at $T_{w,c}$ = 70°F. Both tubes have outer diameter D_o = 0.75 inch and thickness th = 0.065 inch. The conductivity of the tubing material is k_t = 0.22 W/m-K. The heat transfer coefficient between the water and the tube internal surface (in both tubes) is $\overline{h}_w$ = 250 W/m^2-K. The center to center distance between the tubes is w = 1.25 inch and the length of the tubes is L = 20 ft (into the page). The tubes are buried in soil that has conductivity k_s = 0.30 W/m-K.

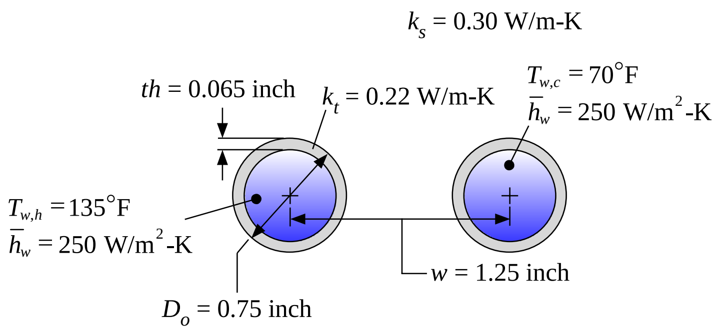

Figure P2.1-1: Tubes buried in soil.

a.) Estimate the heat transfer from the hot water to the cold water due to their proximity to one another.

b.) To do part (a) you should have needed to determine a shape factor; calculate an approximate value of the shape factor and compare it to the accepted value.

c.) Plot the rate of heat transfer from the hot water to the cold water as a function of the center to center distance between the tubes.

2.1-3 (2-2 in text) A solar electric generation system (SEGS) employs molten salt as both the energy transport and storage fluid. The molten salt is heated to 500°C and stored in a buried semi-spherical tank. The top (flat) surface of the tank is at ground level. The diameter of the tank before insulation is applied 14 m. The outside surfaces of the tank are insulated with 0.30 m thick fiberglass having a thermal conductivity of 0.035 W/m-K. Sand having a thermal conductivity of 0.27 W/m-K surrounds the tank, except on its top surface. Estimate the rate of heat loss from this storage unit to the 25°C surroundings.

2.2-3 (2-3 in text) You are the engineer responsible for a simple device that is used to measure heat transfer coefficient as a function of position within a tank of liquid (Figure P2.2-3). The heat transfer coefficient can be correlated against vapor quality, fluid composition, and other useful quantities. The measurement device is composed of many thin plates of low conductivity material that are interspersed with large, copper interconnects. Heater bars run along both edges of the thin plates. The heater bars are insulated and can only transfer energy to the plate; the heater bars are conductive and can therefore be assumed to come to a uniform temperature as a current is applied. This uniform temperature is assumed to be applied to the top and bottom edges of the plates. The copper interconnects are thermally well-connected to the fluid; therefore, the temperature of the left and right edges of each plate are equal to the fluid temperature. This is convenient because it isolates the effect of adjacent plates from one another which allows each plate to measure the local heat transfer coefficient. Both surfaces of the plate are exposed to the fluid temperature via a heat transfer coefficient. It is possible to infer the heat transfer coefficient by measuring heat transfer required to elevate the heater bar temperature a specified temperature above the fluid temperature.

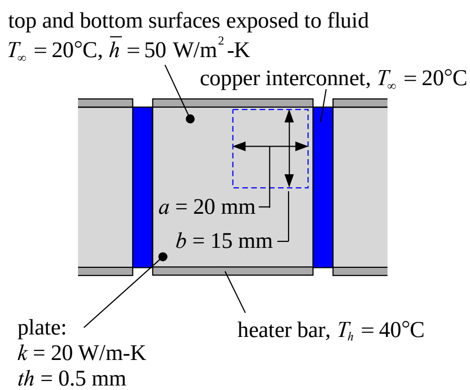

Figure P2.2-3: Device to measure heat transfer coefficient as a function of position.

The nominal design of an individual heater plate utilizes metal with k = 20 W/m-K, th = 0.5 mm, a = 20 mm, and b = 15 mm (note that a and b are defined as the half-width and half-height of the heater plate, respectively, and th is the thickness as shown in Figure P2-3). The heater bar temperature is maintained at T_h = 40ºC and the fluid temperature is T_∞ = 20ºC. The nominal value of the average heat transfer coefficient is $\overline{h}$ = 50 W/m^2-K.

a.) Develop an analytical model that can predict the temperature distribution in the plate under these nominal conditions.

b.) The measured quantity is the rate of heat transfer to the plate from the heater ($\dot{q}_h$) and therefore the relationship between $\dot{q}_h$ and $\overline{h}$ (the quantity that is inferred from the heater power) determines how useful the instrument is. Determine the heater power.

c.) If the uncertainty in the measurement of the heater power is $\delta\dot{q}_h$ = 0.01 W, estimate the uncertainty in the measured heat transfer coefficient ($\delta\overline{h}$).

2.2-6 (2-4 in text) A laminated composite structure is shown in Figure P2.2-6.

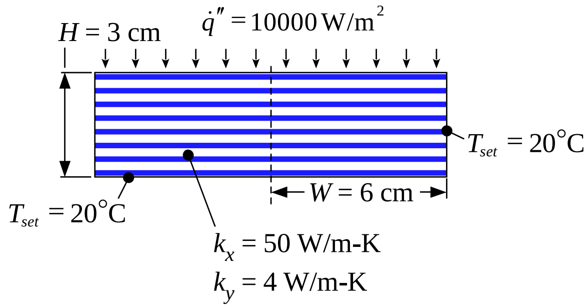

Figure P2.2-6: Composite structure exposed to a heat flux.

The structure is anisotropic. The effective conductivity of the composite in the *x*-direction is k_x = 50 W/m-K and in the *y*-direction it is k_y = 4 W/m-K. The top of the structure is exposed to a heat flux of $\dot{q}''$ = 10,000 W/m^2. The other edges are maintained at T_{set} = 20°C. The height of the structure is H = 3 cm and the half-width is W = 6 cm.

a.) Develop a separation of variables solution for the 2-D steady-state temperature distribution in the composite.

b.) Prepare a contour plot of the temperature distribution.

Section 2.3: Advanced Separation of Variables Solutions

2.3-1 (2-5 in text) Figure P2.3-1 illustrates a pipe that connects two tanks of liquid oxygen on a spacecraft. The pipe is subjected to a heat flux, $\dot{q}''$ = 8,000 W/m^2, which can be assumed to be uniformly applied to the outer surface of the pipe and entirely absorbed. Neglect radiation from the surface of the pipe to space. The inner radius of the pipe is r_{in} = 6 cm, the outer radius of the pipe is r_{out} = 10 cm, and the half-length of the pipe is L = 10 cm. The ends of the pipe are attached to the liquid oxygen tanks and therefore are at a uniform temperature of T_{LOx} = 125 K. The pipe is made of a material with a conductivity of k = 10 W/m-K. The pipe is empty and therefore the internal surface can be assumed to be adiabatic.

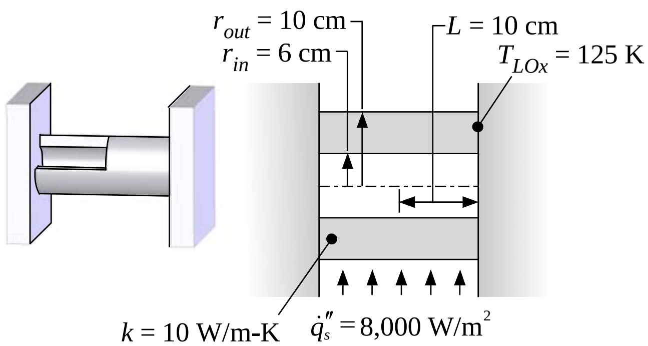

Figure P2.3-1: Cryogen transfer pipe connecting two liquid oxygen tanks.

a.) Develop an analytical model that can predict the temperature distribution within the pipe. Prepare a contour plot of the temperature distribution within the pipe.

2.3-2 (2-6 in text) Figure P2.3-2 illustrates a cylinder that is exposed to a concentrated heat flux at one end.

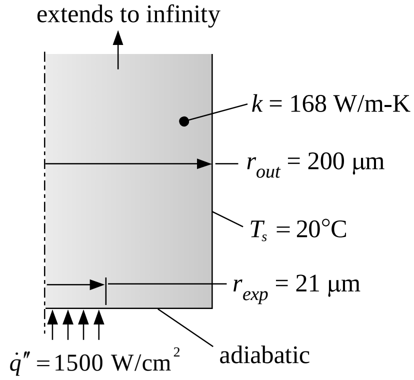

Figure P2.3-2: Cylinder exposed to a concentrated heat flux at one end.

The cylinder extends infinitely in the *x*-direction. The surface at $x = 0$ experiences a uniform heat flux of $\dot{q}'' = 1500\ \text{W/cm}^2$ for $r < r_{exp} = 21\ \mu\text{m}$ and is adiabatic for $r_{exp} < r < r_{out}$ where $r_{out} = 200\ \mu\text{m}$ is the outer radius of the cylinder. The outer surface of the cylinder is maintained at a uniform temperature of $T_s = 20°\text{C}$. The conductivity of the cylinder material is $k = 168$ W/m-K.

a.) Develop a separation of variables solution for the temperature distribution within the cylinder. Plot the temperature as a function of radius for various values of *x*.

b.) Determine the average temperature of the cylinder at the surface exposed to the heat flux.

c.) Define a dimensionless thermal resistance between the surface exposed to the heat flux and T_s. Plot the dimensionless thermal resistance as a function of r_{out}/r_{in}.

d.) Show that your plot from (c) does not change if the problem parameters (e.g., T_s, k, etc.) are changed.

Section 2.4: Superposition

2.4-1 (2-7 in text) The plate shown in Figure P2.4-1 is exposed to a uniform heat flux $\dot{q}'' = 1\text{x}10^5$ W/m^2 along its top surface and is adiabatic at its bottom surface. The left side of the plate is kept at T_L = 300 K and the right side is at T_R = 500 K. The height and width of the plate are H = 1 cm and W = 5 cm, respectively. The conductivity of the plate is k = 10 W/m-K.

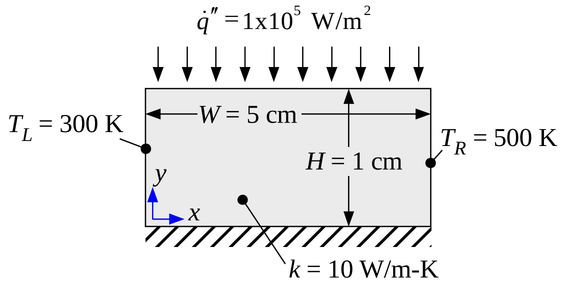

Figure P2.4-1: Plate.

a.) Derive an analytical solution for the temperature distribution in the plate.
b.) Implement your solution in EES and prepare a contour plot of the temperature.

Section 2.5: Numerical Solutions to Steady-State 2-D Problems using EES

2.5-1 (2-8 in text) Figure P2.5-1 illustrates an electrical heating element that is affixed to the wall of a chemical reactor. The element is rectangular in cross-section and very long (into the page). The temperature distribution within the element is therefore two-dimensional, $T(x, y)$. The width of the element is a = 5.0 cm and the height is b = 10.0 cm. The three edges of the element that are exposed to the chemical (at x = 0, y = 0, and x = a) are maintained at a temperature T_c = 200°C while the upper edge (at y = b) is affixed to the well-insulated wall of the reactor and can therefore be considered adiabatic. The element experiences a uniform volumetric rate of thermal energy generation, $\dot{g}'''$ = 1x10^6 W/m^3. The conductivity of the material is k = 0.8 W/m-K.

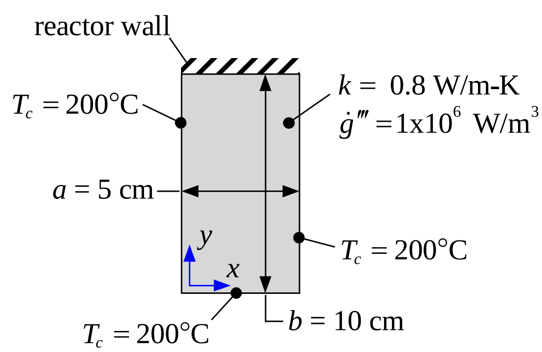

Figure P2.5-1: Electrical heating element.

a.) Develop a 2-D numerical model of the element using EES.

b.) Plot the temperature as a function of x at various values of y. What is the maximum temperature within the element and where is it located?

c.) Prepare a reality check to show that your solution behaves according to your physical intuition. That is, change some aspect of your program and show that the results behave as you would expect (clearly describe the change that you made and show the result).

Section 2.6: Finite-Difference Solutions to Steady-State 2-D Problems using MATLAB

2.6-1 (2-9 in text) Figure P2.6-1 illustrates a cut-away view of two plates that are being welded together. Both edges of the plate are clamped and effectively held at temperatures T_s = 25°C. The top of the plate is exposed to a heat flux that varies with position x, measured from joint, according to: $\dot{q}''_m(x) = \dot{q}''_j \exp(-x/L_j)$ where $\dot{q}''_j$ =1x10^6 W/m^2 is the maximum heat flux (at the joint, x = 0) and L_j = 2.0 cm is a measure of the extent of the heat flux. The back side of the plates are exposed to liquid cooling by a jet of fluid at T_f = -35°C with $\bar{h}$ = 5000 W/m^2-K. A half-symmetry model of the problem is shown in Figure P2.6-1. The thickness of the plate is b = 3.5 cm and the width of a single plate is W = 8.5 cm. You may assume that the welding process is steady-state and 2-D. You may neglect convection from the top of the plate. The conductivity of the plate material is k = 38 W/m-K.

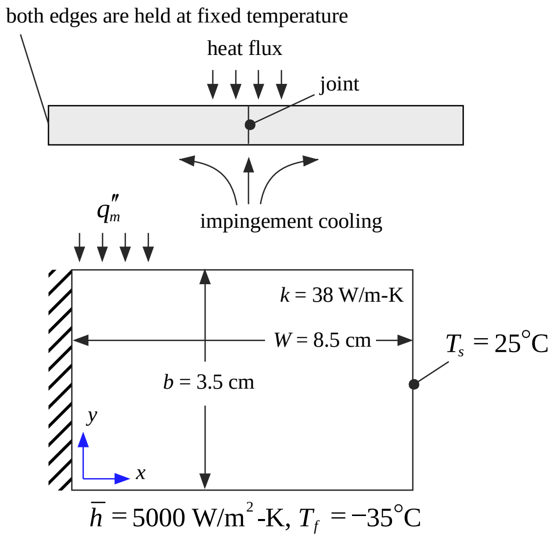

Figure P2.6-1: Welding process and half-symmetry model of the welding process.

a.) Develop a separation of variables solution to the problem (note, this was done previously in Problem 2.2-1). Implement the solution in EES and prepare a plot of the temperature as a function of x at y = 0, 1.0, 2.0, 3.0, and 3.5 cm.

b.) Prepare a contour plot of the temperature distribution.

c.) Develop a numerical model of the problem. Implement the solution in MATLAB and prepare a contour or surface plot of the temperature in the plate.

d.) Plot the temperature as a function of x at y = 0, b/2, and b and overlay on this plot the separation of variables solution obtained in part (a) evaluated at the same locations.

Section 2.7: Finite-Element Solutions to Steady-State 2-D Problems using FEHT

2.7-1 (2-10 in text) Figure P2.7-1(a) illustrates a double paned window. The window consists of two panes of glass each of which is tg = 0.95 cm thick and W = 4 ft wide by H = 5 ft high. The glass panes are separated by an air gap of g = 1.9 cm. You may assume that the air is stagnant with k_a = 0.025 W/m-K. The glass has conductivity k_g = 1.4 W/m-K. The heat transfer coefficient between the inner surface of the inner pane and the indoor air is $\overline{h}_{in}$ = 10 W/m^2-K and the heat transfer coefficient between the outer surface of the outer pane and the outdoor air is $\overline{h}_{out}$ = 25 W/m^2-K. You keep your house heated to T_{in} = 70°F.

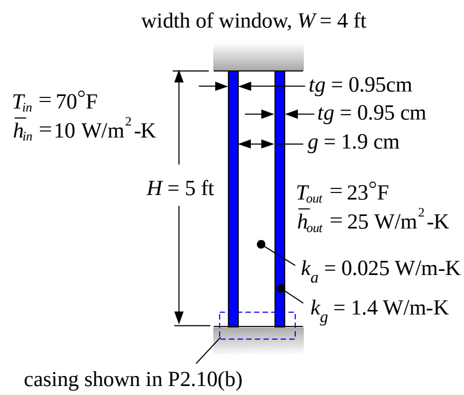

Figure P2.7-1(a): Double paned window.

The average heating season in Madison lasts about $time$ = 130 days and the average outdoor temperature during this time is T_{out} = 23°F. You heat with natural gas and pay, on average, ec = 1.415 \$/therm (a therm is an energy unit =1.055x10^8 J).

a.) Calculate the average rate of heat transfer through the double paned window during the heating season.

b.) How much does the energy lost through the window cost during a single heating season?

There is a metal casing that holds the panes of glass and connects them to the surrounding wall, as shown in Figure P2.7-1(b). Because the metal casing is high conductivity, it seems likely that you could lose a substantial amount of heat by conduction through the casing (potentially negating the advantage of using a double paned window). The geometry of the casing is shown in Figure P2.7-1(b); note that the casing is symmetric about the center of the window.

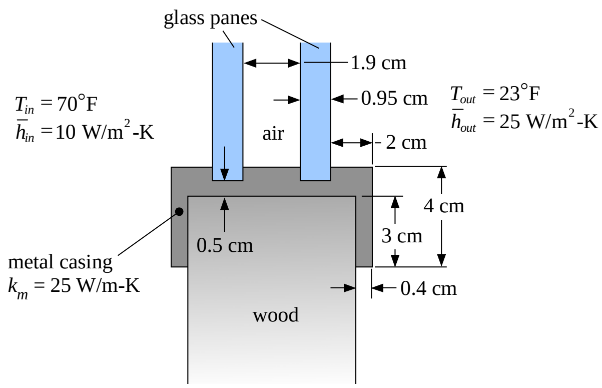

Figure P2-10(b) Metal casing.

All surfaces of the casing that are adjacent to glass, wood, or the air between the glass panes can be assumed to be adiabatic. The other surfaces are exposed to either the indoor or outdoor air.

c.) Prepare a 2-D thermal analysis of the casing using FEHT. Turn in a print out of your geometry as well as a contour plot of the temperature distribution. What is the rate of energy lost via conduction through the casing per unit length (W/m)?

d.) Show that your numerical model has converged by recording the rate of heat transfer per length for several values of the number of nodes.

e.) How much does the casing add to the cost of heating your house?

2.7-3 (2-11 in text) A radiator panel extends from a spacecraft; both surfaces of the radiator are exposed to space (for the purposes of this problem it is acceptable to assume that space is at 0 K); the emittance of the surface is $\varepsilon = 1.0$. The plate is made of aluminum (k = 200 W/m-K and ρ = 2700 kg/m^3) and has a fluid line attached to it, as shown in Figure 2.7-3(a). The half-width of the plate is a=0.5 m wide while the height of the plate is b=0.75m. The thickness of the plate is a design variable and will be varied in this analysis; begin by assuming that the thickness is th = 1.0 cm. The fluid lines carry coolant at T_c = 320 K. Assume that the fluid temperature is constant although the fluid temperature will actually decrease as it transfers heat to the radiator. The combination of convection and conduction through the panel-to-fluid line mounting leads to an effective heat transfer coefficient of h = 1,000 W/m^2-K over the 3.0 cm strip occupied by the fluid line.

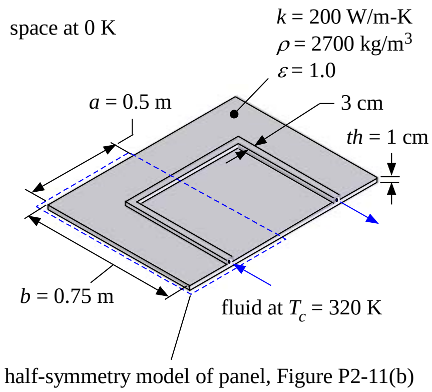

Figure 2.7-3(a): Radiator panel

The radiator panel is symmetric about its half-width and the critical dimensions that are required to develop a half-symmetry model of the radiator are shown in Figure 2.7-3(b). There are three regions associated with the problem that must be defined separately so that the surface conditions can be set differently. Regions 1 and 3 are exposed to space on both sides while Region 2 is exposed to the coolant fluid one side and space on the other; for the purposes of this problem, the effect of radiation to space on the back side of Region 2 is neglected.

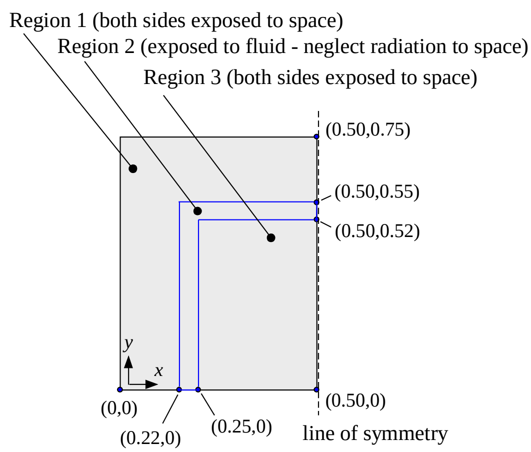

Figure 2.7-3(b): Half-symmetry model.

a.) Prepare a FEHT model that can predict the temperature distribution over the radiator panel.
b.) Export the solution to EES and calculate the total heat transferred from the radiator and the radiator efficiency (defined as the ratio of the radiator heat transfer to the heat transfer from the radiator if it were isothermal and at the coolant temperature).
c.) Explore the effect of thickness on the radiator efficiency and mass.

Section 2.8: Resistance Approximations for Conduction Problems

2.8-1 (2-12 in text) There are several cryogenic systems that require a "thermal switch", a device that can be used to control the thermal resistance between two objects. One class of thermal switch is activated mechanically and an attractive method of providing mechanical actuation at cryogenic temperatures is with a piezoelectric stack; unfortunately, the displacement provided by a piezoelectric stack is very small, typically on the order of 10 microns. A company has proposed an innovative design for a thermal switch, shown in Figure P2.8-1(a). Two blocks are composed of th = 10 μm laminations that are alternately copper (k_{Cu} = 400 W/m-K) and plastic (k_p = 0.5 W/m-K). The thickness of each block is L = 2.0 cm in the direction of the heat flow. One edge of each block is carefully polished and these edges are pressed together; the contact resistance associated with this joint is $R''_c = 5\text{x}10^{-4}$ K-m^2/W.

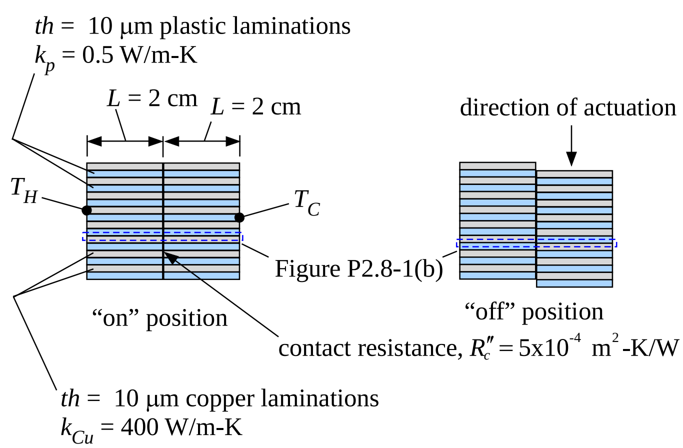

Figure P2.8-1(a): Thermal switch in the "on" and "off" positions.

Figure P2.8-1(a) shows the orientation of the two blocks when the switch is in the "on" position; notice that the copper laminations are aligned with one another in this configuration which provides a continuous path for heat through high conductivity copper (with the exception of the contact resistance at the interface). The vertical location of the right-hand block is shifted by 10 μm to turn the switch "off". In the "off" position, the copper laminations are aligned with the plastic laminations; therefore, the heat transfer is inhibited by low conductivity plastic. Figure P2.8-1(b) illustrates a closer view of half (in the vertical direction) of two adjacent laminations in the "on" and "off" configurations. Note that the repeating nature of the geometry means that it is sufficient to analyze a single lamination set and assume that the upper and lower boundaries are adiabatic.

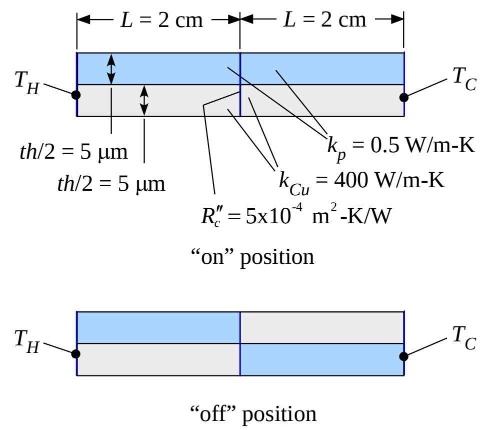

Figure P2.8-1(b): A single set consisting of half of two adjacent laminations in the "on" and "off" positions.

The key parameter that characterizes a thermal switch is the resistance ratio (*RR*) which is defined as the ratio of the resistance of the switch in the "off" position to its resistance in the "on" position. The company claims that they can achieve a resistance ratio of more than 100 for this switch.

a) Estimate upper and lower bounds for the resistance ratio for the proposed thermal switch using 1-D conduction network approximations. Be sure to draw and clearly label the resistance networks that are used to provide the estimates. Use your results to assess the company's claim of a resistance ratio of 100.

b) Provide one or more suggestions for design changes that would improve the performance of the switch (i.e., increase the resistance ratio). Justify your suggestions.

c.) Sketch the temperature distribution through the two parallel paths associated with the adiabatic limit of the switch's operation in the "off" position. Do not worry about the quantitative details of the sketch, just make sure that the qualitative features are correct.

d.) Sketch the temperature distribution through the two parallel paths associated with the adiabatic limit in the "on" position. Again, do not worry about the quantitative details of your sketch, just make sure that the qualitative features are correct.

P2.8-2 (2-13 in text) Figure P2.8-2 illustrates a thermal bus bar that has width W = 2 cm (into the page).

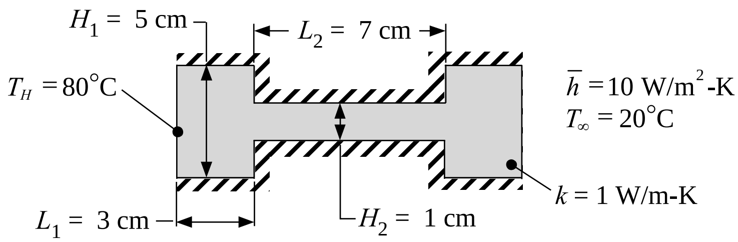

Figure P2.8-2: Thermal bus bar.

The bus bar is made of a material with conductivity k = 1 W/m-K. The middle section is L_2 = 7 cm long with thickness H_2 = 1 cm. The two ends are each L_1 = 3 cm long with thickness H_1 = 3 cm. One end of the bar is held at T_H = 80ºC and the other is exposed to air at T_∞ = 20ºC with $\bar{h}$ = 10 W/m^2-K.

a.) Use FEHT to predict the rate of heat transfer through the bus bar.

b.) Obtain upper and lower bounds for the rate of heat transfer through the bus bar using appropriately defined resistance approximations.

2.9-2 (2-14 in text) A laminated stator is shown in Figure P2.9-2. The stator is composed of laminations with conductivity k_{lam} = 10 W/m-K that are coated with a very thin layer of epoxy with conductivity k_{epoxy} = 2.0 W/m-K in order to prevent eddy current losses. The laminations are th_{lam} = 0.5 mm thick and the epoxy coating is 0.1 mm thick (the total amount of epoxy separating each lamination is th_{epoxy} = 0.2 mm). The inner radius of the laminations is r_{in}= 8.0 mm and the outer radius of the laminations is $r_{o,lam}$ = 20 mm. The laminations are surrounded by a cylinder of plastic with conductivity k_p = 1.5 W/m-K that has an outer radius of $r_{o,p}$ = 25 mm. The motor casing surrounds the plastic. The motor casing has an outer radius of $r_{o,c}$ = 35 mm and is composed of aluminum with conductivity k_c = 200 W/m-K.

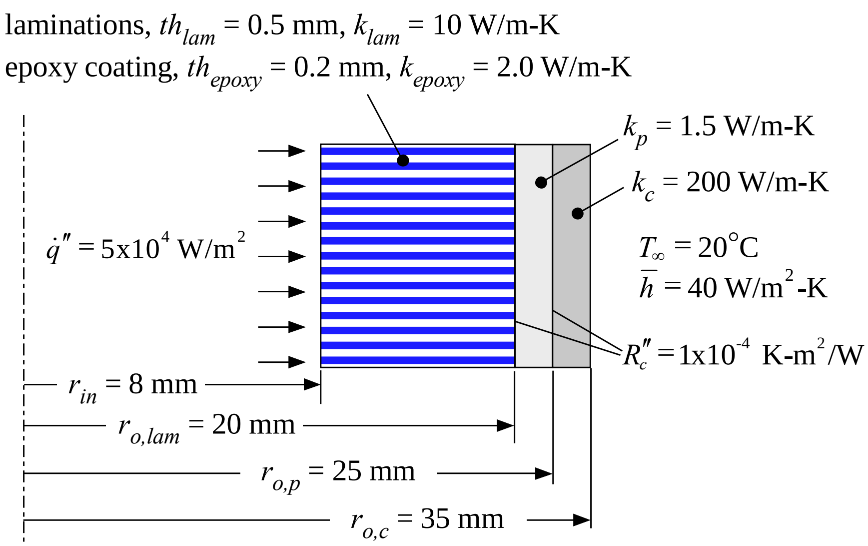

Figure P2.9-2: Laminated stator.

A heat flux associated with the windage loss associated with the drag on the shaft is $\dot{q}''$ = 5x10^4 W/m^2 is imposed on the internal surface of the laminations. The outer surface of the motor is exposed to air at T_∞ = 20°C with a heat transfer coefficient $\bar{h}$ = 40 W/m^2-K. There is a contact resistance R''_c = 1x10^{-4} K-m^2/W between the outer surface of the laminations and the inner surface of the plastic and the outer surface of the plastic and the inner surface of the motor housing.

a.) Determine an upper and lower bound for the temperature at the inner surface of the laminations (T_{in}).

b.) You need to reduce the internal surface temperature of the laminations and there are a few design options available, including: (1) increase the lamination thickness (up to 0.7 mm), (2) reduce the epoxy thickness (down to 0.05 mm), (3) increase the epoxy conductivity (up to 2.5 W/m-K), or (4) increase the heat transfer coefficient (up to 100 W/m-K). Which of these options do you suggest and why?