
Notation

Abbreviations

Ab	category of abelian groups, xv
$\text{Ab}(\mathcal{A})$	abelian category associated with a locally finitely presented category \mathcal{A} , 384
$\text{Ab}(X)$	abelian category associated with an object X of a locally finitely presented category, 392
$\text{Ab}(\Lambda)$	free abelian category over a ring Λ , 400
$\mathbf{Ac}(\mathcal{A})$	acyclic complexes in an exact category \mathcal{A} , 107
$\text{add}(\mathcal{X})$	closure of \mathcal{X} under finite direct sums and summands, xxi
$\text{Add}(\mathcal{X})$	closure of \mathcal{X} under all coproducts and direct summands, xxiii
$\text{Add}(\mathcal{C}, \text{Ab})$	additive functors $\mathcal{C} \rightarrow \text{Ab}$, 345
$\text{Ann}(X)$	annihilator of a module X , 48
$\text{Ass}(X)$	associated prime ideals of a module X , 50
$\mathbf{C}(\mathcal{A})$	complexes in an additive category \mathcal{A} , 102
$\mathbf{C}^b(\mathcal{A})$	bounded complexes in \mathcal{A} , 109
$\mathbf{C}^+(\mathcal{A})$	bounded below complexes in \mathcal{A} , 109
$\mathbf{C}^-(\mathcal{A})$	bounded above complexes in \mathcal{A} , 109
$\text{card}(X)$	cardinality of a set X , xv
$\text{coh}(\mathbb{X})$	coherent sheaves on a scheme \mathbb{X} , 344
$\text{Coker}(\phi)$	cokernel of a morphism ϕ , xxii
$\text{colim}(F)$	colimit of a functor F , xix
$\text{Cone}(\phi)$	cone of a morphism ϕ , 76
$\text{Cores}(\mathcal{C})$	objects that admit a finite coresolution in \mathcal{C} , 208
$\mathbf{D}(\mathcal{A})$	derived category of an exact category \mathcal{A} , 107
$\mathbf{D}^b(\mathcal{A})$	derived category of bounded complexes in \mathcal{A} , 109
$\mathbf{D}^+(\mathcal{A})$	derived category of bounded below complexes in \mathcal{A} , 109

$\mathbf{D}^-(\mathcal{A})$	derived category of bounded above complexes in \mathcal{A} , 109
$\mathbf{D}(\mathcal{A}, \mathcal{A}_0)$	derived category of a Frobenius pair $(\mathcal{A}, \mathcal{A}_0)$, 88
$\mathbf{D}(A)$	derived category of a dg algebra A , 299
$\mathbf{D}^{\text{perf}}(A)$	perfect complexes of a ring or dg algebra A , 167, 300
$\mathbf{D}_{\text{sg}}(A)$	singularity category of a ring A , 182
$D(X)$	Matlis dual of a module X , xxxi
Δ_i, ∇_i	(co)standard module over a quasi-hereditary algebra, 232
$E(X)$	injective envelope of an object X , xxviii
$\text{eff}(\mathcal{A})$	effaceable functors $\mathcal{A}^{\text{op}} \rightarrow \text{Ab}$, 42
$\text{Eff}(\mathcal{A})$	locally effaceable functors $\mathcal{A}^{\text{op}} \rightarrow \text{Ab}$, 44
$\text{Eff}(\mathcal{A}, \text{Ab})$	effaceable functors $\mathcal{A} \rightarrow \text{Ab}$, 399
$\text{End}(X)$	endomorphisms of an object X , xvii
$\text{End}(X)$	endomorphism dg algebra of X , 299
$\text{endol}(X)$	endolength of a module X , 426
Ess	essential monomorphisms in an abelian category, 59
$\text{Ex}(\mathcal{C}, \text{Ab})$	exact functors $\mathcal{C} \rightarrow \text{Ab}$, 353
$\text{Ext}^n(X, Y)$	degree n extensions between objects X, Y , xxv
$\widehat{\text{Ext}}^n(X, Y)$	degree n Tate extensions between objects X, Y , 143
$\mathcal{F}(\Lambda)$	finitely generated free modules over a ring Λ , 375
$\text{Filt}(\mathcal{X})$	extension closed subcategory generated by a class \mathcal{X} , xxvi
$\text{fp}(\mathcal{A})$	finitely presented objects of a category \mathcal{A} , 343
$\text{Fp}(\mathcal{C}, \text{Ab})$	finitely presented functors $\mathcal{C} \rightarrow \text{Ab}$, 346
$\text{Fun}(\mathcal{C}, \mathcal{D})$	functors $\mathcal{C} \rightarrow \mathcal{D}$, xviii
$\text{GI}(X)$	Gorenstein injective approximation of a module X , 193
$\text{Ginj}(\Lambda)$	Gorenstein injective modules over a ring Λ , 192
$\overline{\text{Ginj}}(\Lambda)$	stable category of Gorenstein injective modules, 193
$\text{GL}_n(\Lambda)$	general linear group of $n \times n$ matrices over a ring Λ , 375
$\text{gl.dim}(\mathcal{A})$	global dimension of an exact category \mathcal{A} , xxix
$\text{Gor.dim}(\Lambda)$	dimension of a Gorenstein ring Λ , 184
$\text{GP}(X)$	Gorenstein projective approximation of a module X , 193
$\text{Gproj}(\Lambda)$	Gorenstein projective modules over a ring Λ , 179
$\overline{\text{Gproj}}(\Lambda)$	stable category of Gorenstein projective modules, 182
$\overline{\text{GrMod}}(\Lambda)$	graded modules over a graded ring Λ , 54
$\text{grmod}(\Lambda)$	finitely presented graded modules, 54
$\underline{\text{grmod}}(\Lambda)$	projectively stable module category, 54
$\text{grproj}(\Lambda)$	finitely generated projective graded modules, 54
Γ	category of finite sets, 373
Γ_{inj}	category of finite sets with injective morphisms, 375
Γ_{os}	category of finite sets with ordered surjections, 373

Γ_{sur}	category of finite sets with surjective morphisms, 373
$\Gamma^*(V)$	algebra of symmetric tensors of a module V , 244
$\Gamma^d\mathcal{P}_k$	category of symmetric tensors over k , 244
h_λ	complete symmetric function for a partition λ , 243
$H^n(X)$	cohomology of degree n of a complex X , 104
$H^n(G, X)$	cohomology of degree n of a group G with coefficients in a module X , 408
$\hat{H}^n(G, X)$	Tate cohomology of degree n of a group G with coefficients in a module X , 411
$\text{hocolim}(X_n)$	homotopy colimit of a sequence $(X_n \rightarrow X_{n+1})_{n \in \mathbb{N}}$, 90
$\text{holim}(X_n)$	homotopy limit of a sequence $(X_{n+1} \rightarrow X_n)_{n \in \mathbb{N}}$, 90
$\text{Hom}(X, Y)$	set (or complex) of morphisms $X \rightarrow Y$, xvii, 129
$\overline{\text{Hom}}(X, Y)$	stable morphisms modulo injectives, 190
$\underline{\text{Hom}}(X, Y)$	stable morphisms modulo projectives, 83, 190
$\mathcal{H}om(\mathcal{C}, \mathcal{D})$	functors $\mathcal{C} \rightarrow \mathcal{D}$, xviii
$\mathcal{H}om(X, Y)$	dg module of morphisms $X \rightarrow Y$, 298
$\text{ht}(X)$	height of an object X , xxiv
$i(X)$	injective resolution of an object X , 112
$\mathbf{i}(X)$	K-injective resolution of a complex X , 123
id_X	identity morphism of an object X , xvii
$\text{id}_{\mathcal{C}}$	identity functor of a category \mathcal{C} , xvii
$\text{Im}(\phi)$	image of a morphism ϕ , xxii
$\text{Im}(F)$	essential image of a functor F , xviii
$\text{ind}(\mathcal{A})$	indecomposable objects of a Krull–Schmidt category \mathcal{A} , 422
$\text{Ind}(\mathcal{A})$	indecomposable pure-injective objects of a locally finitely presented category \mathcal{A} , 384
$\text{Ind}(\Lambda)$	indecomposable pure-injective modules over a ring Λ , 400
$\text{Inj}(\mathcal{A})$	injective objects of an exact category \mathcal{A} , xxviii
$\text{Inj}(\Lambda)$	injective modules over a ring Λ , 23
$\text{inj}(\Lambda)$	finitely presented injective modules, 195
$\text{inj.dim}(X)$	injective dimension of an object X , xxix
$J(\Lambda)$	Jacobson radical of a ring Λ , xxiv
$K_{\lambda\mu}$	Kostka number for partitions λ, μ , 242
$K_0(\mathcal{C})$	Grothendieck group of an exact or triangulated category \mathcal{C} , xxx, 110
$K_0(\Lambda)$	Grothendieck group of a ring Λ , xxx
$\mathbf{K}(\mathcal{A})$	homotopy category of complexes in an additive category \mathcal{A} , 103

$\mathbf{K}^b(\mathcal{A})$	homotopy category of bounded complexes in \mathcal{A} , 109
$\mathbf{K}^+(\mathcal{A})$	homotopy category of bounded below complexes in \mathcal{A} , 109
$\mathbf{K}^-(\mathcal{A})$	homotopy category of bounded above complexes in \mathcal{A} , 109
$\mathbf{K}^{+,b}(\mathcal{C})$	homotopy category of bounded below complexes with bounded cohomology, 114
$\mathbf{K}^{-,b}(\mathcal{C})$	homotopy category of bounded above complexes with bounded cohomology, 114
$\mathbf{K}_{\text{ac}}(\mathcal{P})$	acyclic complexes of projectives in a Frobenius category, 142
$\mathbf{K}_{\text{inj}}(\mathcal{A})$	K-injective complexes in an exact category \mathcal{A} , 122
$\mathbf{K}_{\text{proj}}(\mathcal{A})$	K-projective complexes in an exact category \mathcal{A} , 122
$\text{Ker}(\phi)$	kernel of a morphism ϕ , xxii
$\text{Ker}(F)$	kernel of a functor F , xxi
$\text{KG.dim}(\mathcal{A})$	Krull–Gabriel dimension of an abelian category \mathcal{A} , 436
$\text{KG.dim}(X)$	Krull–Gabriel dimension of an object X , 441
$\mathbf{L}F$	left derived functor of a functor F , 128
$\mathbf{L}(X)$	lattice of subobjects of an object X , xxxi
$\ell(X)$	composition length of an object X , xxiv
$\text{Lex}(\mathcal{A})$	left exact functors $\mathcal{A}^{\text{op}} \rightarrow \text{Ab}$ for an exact category \mathcal{A} , 44
$\text{Lex}(\mathcal{C}^{\text{op}}, \text{Ab})$	left exact functors $\mathcal{C}^{\text{op}} \rightarrow \text{Ab}$ for an additive category \mathcal{C} with cokernels, 349
$\lim(F)$	limit of a functor F , xix
$\text{Loc}(\mathcal{X})$	localising subcategory generated by a class \mathcal{X} , 92
$\Lambda^*(V)$	exterior algebra of a module V , 246
$\Lambda(n, d)$	compositions of d into n parts, 241
$M_n(\Lambda)$	semigroup of $n \times n$ matrices over a ring Λ , 375
$\text{Max}(\Lambda)$	maximal ideals of a commutative ring Λ , 445
$\text{m.dim}(L)$	m-dimension of a lattice L , 439
$\text{Mod}(\mathcal{C})$	additive functors $\mathcal{C}^{\text{op}} \rightarrow \text{Ab}$, 16
$\text{mod}(\mathcal{C})$	finitely presented functors $\mathcal{C}^{\text{op}} \rightarrow \text{Ab}$, 17
$\text{mod}_{\alpha}(\mathcal{C})$	α -presentable functors $\mathcal{C}^{\text{op}} \rightarrow \text{Ab}$, 65
$\text{Mod}(\Lambda)$	modules over a ring Λ , xv
$\underline{\text{mod}}(\Lambda)$	finitely presented modules, xv
$\overline{\text{mod}}(\Lambda)$	injectively stable module category, 191
$\underline{\underline{\text{mod}}}(\Lambda)$	projectively stable module category, 43
$\text{Mor}(\mathcal{C})$	morphisms of a category \mathcal{C} , xvii
\mathbb{N}	set of non-negative integers, xvi
$\vec{\mathbb{N}}, \vec{\vec{\mathbb{N}}}$	category of non-negative integers, 367
$\text{noeth}(\mathcal{A})$	noetherian objects of an abelian category \mathcal{A} , 37

ν	Nakayama functor, 195
$\mathcal{O}_{\mathbb{X}}$	structure sheaf of a scheme \mathbb{X} , 152
$\text{Ob}(\mathcal{C})$	objects of a category \mathcal{C} , xvii
$\Omega(X)$	syzygy of a module X , xxix
$\Omega_{\mathbb{X}/k}$	sheaf of differential forms of \mathbb{X} over k , 318
\mathbb{P}_k^n	projective n -space over a field k , 329
$\mathbf{P}(\mathcal{A})$	purity category of a locally finitely presented category \mathcal{A} , 379
$\mathcal{P}(\Lambda)$	modules that admit a finite resolution in $\text{proj } \Lambda$, 218
\mathcal{P}_k	finitely generated projective k -modules, 243
$p(X)$	projective resolution of an object X , 112
$\mathbf{p}(X)$	\mathbf{K} -projective resolution of a complex X , 123
$\text{pcoh}(\Lambda)$	pseudo-coherent modules over a ring Λ , 171
$\text{Ph}(X, Y)$	phantom morphisms $X \rightarrow Y$, 167
$\text{Pol}^d \mathcal{P}_k$	strict polynomial functors of degree d over k , 251
$\text{pol}^d \mathcal{P}_k$	finite strict polynomial functors of degree d over k , 251
$\text{Prod}(X)$	closure of X under all products and direct summands, 395
$\text{Proj}(\mathcal{A})$	projective objects of an exact category \mathcal{A} , xxviii
$\text{Proj}(\Lambda)$	projective modules over a ring Λ , 23
$\text{proj}(\Lambda)$	finitely generated projective modules, xv
$\text{proj.dim}(X)$	projective dimension of an object X , xxviii
$\text{Qcoh}(\mathbb{X})$	quasi-coherent sheaves on a scheme \mathbb{X} , 344
Qis	quasi-isomorphisms of complexes in an exact category, 107
$\text{rad}(X)$	radical of an object X , xxiv
$\text{Rad}(X, Y)$	group of radical morphisms $X \rightarrow Y$, xxiv
$\text{rank}_k(X)$	rank of a free k -module X , xxiv
$\text{reg}(\Lambda)$	regular modules over an Artin algebra Λ , 158
$\text{rep}(\Gamma, k)$	finite k -linear representations of Γ , 285
$\text{Rep}(\Gamma, k)$	k -linear representations of Γ , 348
$\text{Res}(\mathcal{C})$	objects that admit a finite resolution in \mathcal{C} , 177
$\mathbf{R}F$	right derived functor of a functor F , 128
$\mathbf{R}\text{Hom}(X, Y)$	derived hom of complexes X and Y , 130
$\mathbf{R}\text{lim } F$	right derived limit of a functor F , 321
s_λ	Schur function for a partition λ , 243
$S^*(V)$	symmetric algebra of a module V , 244
Sch^λ	Schur functor for a partition λ , 279
Set	category of sets, xv
$\text{sgn}(\sigma)$	signum of a permutation σ , 279
$\text{Sh}(X)$	sheaves on a topological space X , 35
$\text{soc}(X)$	socle of an object X , xxiv

$\text{span}_k(X)$	k -linear span of a set X , 265
$\text{Sp}(\mathcal{A})$	spectrum of a Grothendieck category \mathcal{A} , 37
$\text{Spec}(A)$	prime ideal spectrum of a commutative ring A , 48
$S_k(n, d)$	Schur algebra over k given by parameters n, d , 245
$S(\mathbf{p}, \lambda)$	coordinate algebra of a weighted projective line, 334
$\text{Sq}(\mathbf{p}, \lambda)$	squid algebra given by a weighted projective line, 336
$\text{St}(\mathcal{A})$	injectively stable category of an exact category \mathcal{A} , 28, 83
$\text{StMod}(\Lambda)$	stable module category of a quasi-Frobenius ring Λ , 88
$\text{Sub}(X)$	poset of subobjects of an object X , 370
$\text{Supp}(F)$	support of a functor F , 422
$\text{Supp}(\mathcal{F})$	support of a sheaf \mathcal{F} , 153
$\text{Supp}(X)$	support of a module or complex X , 48, 162, 133
SW_f	Spanier–Whitehead category of finite CW-complexes, 308
\mathfrak{S}_d	symmetric group, 244
\mathfrak{S}_λ	Young subgroup of the symmetric group, 247
$\Sigma(X)$	suspension or shift of an object X , 73
$\sigma_{\leq n}X, \sigma_{\geq n}X$	brutal truncations of a complex X , 111
$t(X)$	complete resolution of an object X , 143
$T(A)$	trivial extension algebra of an Artin algebra A , 164
$T^*(V)$	tensor algebra of a module V , 246
$\text{Thick}(\mathcal{X})$	thick subcategory generated by a class of objects \mathcal{X} , xxvii, 77
$\text{top}(X)$	top of an object X , xxiv
$\text{Tor}_n^\Lambda(X, Y)$	Tor group of degree n of modules X and Y , 130
$\text{Tr}(X)$	transpose of a finitely presented module X , 191
$\tau_{\leq n}X, \tau_{\geq n}X$	soft truncations of a complex X , 111
$\mathcal{V}(\mathfrak{a})$	prime ideals containing an ideal \mathfrak{a} , 48
$\text{vect}(\mathbb{X})$	vector bundles on a scheme \mathbb{X} , 317
Weyl^λ	Weyl functor for a partition λ , 279
$\text{w.dim}(X)$	weak dimension of a module X , 179
\mathbb{Z}	set of integers, xvi
$Z(\mathcal{C})$	centre of a preadditive category \mathcal{C} , xxx, 347
$Z(\Lambda)$	centre of a ring Λ , xxx
$Z^n(X)$	cocycles of degree n of a complex X , 104

Constructions

A^{op}	opposite of a ring A , xv
A_Σ	universal localisation of a ring A with respect to Σ , 46
A^\dagger	quadratic dual of an algebra A , 331

\mathcal{A}^α	α -presentable objects of a cocomplete category \mathcal{A} , 61
$\mathcal{A}_1 \times_{\mathcal{A}} \mathcal{A}_2$	pullback of abelian categories, 41
$\mathcal{C}^{\mathcal{J}}$	diagrams of type \mathcal{J} in a category \mathcal{C} , xix
\mathcal{C}^{op}	opposite of a category \mathcal{C} , xvii
\mathcal{C}^2	morphisms in a category \mathcal{C} , xvii
$\mathcal{C}(x, y)$	morphisms $x \rightarrow y$ in a category \mathcal{C} , 369
$\mathcal{C}(x)$	morphisms terminating at x in a category \mathcal{C} , 369
$\bar{\mathcal{C}}$	closure of \mathcal{C} under filtered colimits, 345
$\coprod \mathcal{C}$	closure of \mathcal{C} under all coproducts, 97
$(\mathcal{C}_\alpha)_\alpha$	Krull–Gabriel filtration of an abelian category \mathcal{C} , 436
\mathcal{C}/\mathcal{D}	quotient of an additive, abelian, or triangulated category \mathcal{C} with respect to a subcategory $\mathcal{D} \subseteq \mathcal{C}$, 29, 30, 78
\mathcal{C}/F	slice category of \mathcal{C} over a functor F , 345
\mathcal{C}/X	slice category of \mathcal{C} over an object X , 343
$\mathcal{C}[S^{-1}]$	localisation of a category \mathcal{C} with respect to S , 3
$S^{-1}\mathcal{C}$	category of left fractions of \mathcal{C} with respect to S , 10
$\mathcal{C} = \coprod_{i \in I} \mathcal{C}_i$	orthogonal decomposition of an additive category \mathcal{C} , xxi
$\mathcal{C} = \bigvee_{i \in I} \mathcal{C}_i$	direct decomposition of an additive category \mathcal{C} , xxi
$\mathcal{C}^\perp, {}^\perp\mathcal{C}$	perpendicular categories in an abelian, triangulated, exact, or locally finitely presented category, 30, 77, 176, 385
F_λ, F_ρ	left and right adjoint of a functor F , xix
F^λ	λ -fold tensor product of a graded functor F^* , 254
F^\vee	dual of a functor F , 43, 401
F°	dual of a functor F , 243
$k\mathcal{C}$	linearisation of a category \mathcal{C} over a commutative ring, 348
kG	group algebra of a group G over a commutative ring, 348
kQ	path category of a quiver Q over a commutative ring, 348
$k[X]$	free module with basis X over a commutative ring, 348
k_{sgn}	sign representation of the symmetric group, 275
$(L_\alpha)_\alpha$	minimal cofiltration of a modular lattice L , 439
$\lambda \vdash d$	partition of an integer d , 241
S^\perp	class of S -local objects with respect to morphisms in S , 4
X^*	dual of a module X , 180
X^\vee	dual of a module X , 243
X^G	invariants of a module X with G -action, 244
X_G	coinvariants of a module X with G -action, 244
$X(V)_\lambda$	weight space of $X(V)$ for a composition λ , 260
X_ϕ	subgroup of finite definition of $\text{Hom}(C, X)$ with respect to $\phi: C \rightarrow C'$, 363, 392

$X^{(I)}, X[I]$	coproduct of copies of X indexed by a set I , xxiii
$X \otimes_{\Lambda} Y$	tensor product of (complexes of) modules X and Y , xxxiii, 129
$X \otimes_{\Lambda}^L Y$	derived tensor product of complexes X and Y , 130
$\mathcal{X} * \mathcal{Y}$	extensions in a triangulated category of objects in \mathcal{X}, \mathcal{Y} , 97

Arrows

$X \hookrightarrow Y$	monomorphism, xvii
$X \twoheadrightarrow Y$	epimorphism, xvii
$X \xrightarrow{\sim} Y$	isomorphism, xvii
$\mathcal{C} \twoheadrightarrow \mathcal{D}$	fully faithful functor, xviii
$\mathcal{C} \twoheadrightarrow \mathcal{D}$	quotient functor, xviii
$\mathcal{C} \xrightarrow{\sim} \mathcal{D}$	equivalence, xviii
$\mathcal{C} \rightleftarrows \mathcal{D}$	adjoint pair of functors, xviii